

## **Finance and the Preservation of Wealth**

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### Abstract

We introduce the model of asset management developed in Gennaioli, Shleifer, and Vishny (GSV, 2014) into a Solow-style neoclassical growth model with diminishing returns to capital. Savers rely on trusted intermediaries to manage their wealth (claims on capital stock), who can charge fees above costs to trusting investors. In this model, the ratio of financial income to GDP increases with the ratio of aggregate wealth to GDP. Both rise along the convergence path to steady state growth. We examine several further implications of the model for management fees, unit costs of finance, and the consequences of shocks to trust and to the capital stock.

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## **1.Introduction.**

Philippon (2013) documents the astonishing rise of the share of GDP coming from the financial sector since World War II (Figure 1). Financial income rose from about 2% of the total in the 1940s to close to 8% at the time of the financial crisis. Philippon and Reshef (2013) document similar trends in many other developed countries. Greenwood and Scharfstein (2013) show further that, at least in the last 30 years, much of this rise of finance in the United States comes from financial services to consumers, especially asset management and credit intermediation of mortgages and consumer loans.

The growth of the financial sector has proved difficult to explain. Perhaps productivity in finance, as in other services, does not grow as fast as that in other sectors, so we see a manifestation of the Baumol (1967) disease. However, finance has grown relative to other services (Philippon and Reshef 2013), and wages in finance have grown faster than those in other service sectors (Philippon and Reshef 2012), inconsistent with this view. Philippon (2013) treats the cost of finance as a share of intermediated wealth due to screening and monitoring, but does not explain what determines this share or why financial income rises with market wealth. We present a new model of how financial income is endogenously determined as a function of intermediated wealth, describe what shapes this function, and explain how wealth and financial income move together.

Ours is a Solow-style growth model with a financial sector delivering asset management services to savers. A key component of these services is wealth preservation: financial intermediaries enable investors to preserve their savings for future consumption. In doing so, financial intermediaries also enable investors to access investments that make their wealth grow over time on average. As a byproduct of serving investors, intermediaries also provide investment resources to firms. We assume that investors need financial intermediaries to take advantage of these opportunities. On their own they only utilize highly inefficient self-storage, such as keeping money in mattresses or building houses from current income over decades without borrowing funds.

Intermediaries offer savers access to financial services, such as mutual funds or mortgages, which they do not have otherwise. In GSV (2014), we refer to the intermediaries providing such services, be they bankers, brokers, wealth planners, or money managers, as “money doctors.” The analogy captures the idea that even though generic investing in risky assets seems straightforward to economists and finance professors, it actually requires knowledge and confidence that most savers do not have. Savers rely on intermediaries to help them with financial decisions.

But how do investors choose intermediaries? The central assumption of the model is that investors feel less anxious, and therefore better off, investing through intermediaries they trust. The centrality of trust in financial intermediation can be seen from financial advertising, which typically points to experience, trustworthiness, reliability, and even longevity of the intermediaries to attract investors. Guiso, Sapienza, and Zingales (2004, 2008) have pioneered empirical work showing how trust, both across investors and across countries, shapes wealth allocations to risky investment. In our model, intermediaries competitively set their fees to attract clients, but because some intermediaries have a “locational” advantage of being especially trusted by some clients, in equilibrium they charge positive fees that capture a share of expected returns on investments.

GSV (2014) show that this simple model explains a range of puzzling facts about financial services. Their model explains why financial advisors are hired by investors even though they consistently underperform passive investment strategies net of fees, a major puzzle in financial economics since Jensen (1968). It explains why management fees are higher for riskier financial products that have higher expected returns. It explains why money managers pander to investor beliefs when some assets are mispriced. Here we study the aggregate implications of that model for the size of the financial sector, its costs, and their movement over time in response to shocks.

To this end, we embed a version of GSV (2014) into a Solow-style model of capital accumulation and growth under the neoclassical assumption of diminishing returns to capital. In our model, finance income tracks wealth precisely because one of its main functions is to preserve

the stock of wealth, and not just to finance new value added. In addition, we examine the response of the financial sector to shocks to productivity and trust. Finally, we present two extensions of the model helpful for understanding the evidence: free entry of intermediaries and population growth.

This analysis yields several principal implications. First, because the finance income share rises with the ratio of wealth to GDP, the share of finance income in GDP increases over time. The reason is that, with diminishing returns to capital, there are fewer and fewer profitable projects for investing new capital along the convergence path to the steady state. As a consequence, the capital (or wealth) to GDP ratio rises, as does the finance share. Consistent with this analysis, Piketty and Zucman (2014) show that, in recent decades, the wealth to GDP ratio has increased in several advanced economies, including the U.S. We confirm that the U.S. market wealth to GDP ratio has risen over the relevant period. Piketty and Zucman (2014) explain this increase in part by slowdown in total GDP growth over time, which is precisely the mechanism we stress in our setup.

Second, our model sheds light on the evolution of costs of finance. In our model, unit fees on a given financial product fall over time because expected returns to capital fall but also because of increased competition from entry by financial intermediaries. Our model also delivers the prediction that because entry brings investors “closer” to their advisors, they take more risk over time, which might raise the unit costs of finance, since the fees on riskier investments are higher.

These predictions as well find some support in the data. With respect to fees for a given financial product, French (2008) and Greenwood and Scharfstein (2013) find that management fees on equity mutual funds have fallen over time. At the same time, Philippon (2013) shows that unit costs of finance have not fallen, and Greenwood and Scharfstein (2013) document that overall fees, including those on private equity and hedge funds, have stayed roughly constant. Greenwood and Scharfstein (2013) further show that income of financial intermediaries from money management has shifted toward riskier products. These findings are consistent with our model’s predictions. Indeed, we present new evidence that over time both the share of risky assets in the market portfolio

and the share of US population holding stocks have increased, paralleling the growth of finance share. Our model might thus help reconcile the French (2008) evidence on the declining mutual fund fees with Philippon's (2013) finding that unit costs of finance have not fallen: the reason is that investors are taking more risk at higher fees.

Third, our model ties fluctuations in the size of the financial sector to shocks in productivity and trust. In particular, our model predicts that shocks to trust immediately reduce the size of the financial sector, as investors pull resources away from their advisors. Although we do not have a model of endogenous trust determination, and hence cannot make any causal statements, some circumstantial evidence is consistent with this analysis as well. Aghion et al. (2010), Stevenson and Wolfers (2011), Sapienza and Zingales (2012), and Guiso (2010) all present evidence of sharp declines in both generalized trust and trust in the financial system during economic and financial crises. The prolonged decline of the finance share starting in the Great Depression, seen in Figure 1, might be explained in part by declines in trust in the aftermath of the economic collapse.

In Section 2, we describe the model. Section 3 presents the equilibrium in the financial sector. Section 4 considers the full equilibrium in the growth model. Section 5 extends the model to treat endogenous entry and population growth. Section 6 summarizes the empirical implications of the model and puts together some existing and some new evidence. Section 7 concludes. Online Appendices contain all proofs but also several extensions, including asset price bubbles.

## **2. The Model**

### **2.1 The Household Sector**

The economy is inhabited by overlapping generations of young and old. Time starts at  $t = 0$  and goes on forever. A generation born at time  $t - 1$  contains a continuum of workers of size

one, indexed by  $i \in I_{t-1} \equiv [0,1]$ . At  $t - 1$ , during their young age, these workers inelastically supply their unit labor endowments at the equilibrium wage  $w_{t-1}$ . The entire wage income is saved and invested as described below, and consumption takes place only in old age after investment income is received. At the end of  $t$ , the old generation dies without bequest. We begin our analysis by considering an economy with no population growth or technical progress. This simplification allows us to focus on the money management sector, which is the novel part of our analysis. Population growth and technical progress then affect the financial sector only indirectly, by shaping the dynamics of the per capita capital stock and the steady state capital to GDP ratio. Section 5.2 considers population growth, while Online Appendix B.1 considers technical progress.

Workers can invest their resources in two ways. First, they can invest in self-storage. Each unit stored at  $t - 1$  yields  $\gamma \leq 1$  units at  $t$ , so that  $1 - \gamma$  is lost in depreciation. We think of storage as an inefficient way to save on one's own, perhaps by holding cash or gold at home, vulnerable to the risk of theft or inflation. The case of  $\gamma = 1$  captures a perfect self-storage technology. Second, a worker can hire a financial intermediary, whom we refer to as a money manager throughout, to invest his savings in a risky financial asset. At the beginning of time  $t$ , the money manager transforms a worker's savings (one for one) into capital, and rents it to firms, which use it to produce output at the end of time  $t$ . We later describe production in detail.

In the model, we draw a sharp distinction between self-storage, which requires no intermediation, and risky investments, which require money managers. Self-storage can refer to keeping cash in a mattress, or to building a home slowly, over years or decades, without mortgages or loans, as a form of saving (very common in developing countries). Intermediated assets are most naturally thought of as equities, but in a more general setup can include other investments. In reality, the gradation between self-storage and full financial intermediation is more continuous, from cash in mattresses, to bank savings and mortgages, to liquid market investments, to illiquid

investments such as private equity and hedge funds, with increasing amounts of intermediary attention (and cost). Our sharp differentiation is a simplifying assumption.

There are a discrete number  $m > 1$  of money managers in each generation, randomly selected from the young. A generic money manager active at time  $t$  is indexed by  $j \in I_t$ . This money manager charges his investors a profit-maximizing fee  $f_{jt}$  per unit of investment. At time  $t$  all managers invest in the same asset, which yields a stochastic gross return  $R_t$  with mean  $\mathbb{E}\{R_t\}$  and variance  $\sigma_t$ , both of which are determined endogenously in equilibrium. A worker/saver born at time  $t - 1$  delegating at the beginning of time  $t$  his risky investment to manager  $j$  thus earns a net return  $R_t - f_{jt}$ . If the income share invested at time  $t$  in the risky asset is  $\theta_t$ , the worker's consumption in old age is given by:

$$c_{it} = w_{t-1} \cdot [\gamma + \theta_t \cdot (R_t - \gamma - f_{jt})].$$

Consumption increases in the excess return that risky financial assets earn relative to storage (net of the management fee). We impose the constraint  $\theta_t \in (0,1)$  – which in Proposition 1 we verify to hold in equilibrium – because we are interested in cases where risk taking is interior.

After receiving the wage  $w_{t-1}$  at the end of period  $t - 1$ , worker  $i \in I_{t-1}$  chooses at the beginning of time  $t$  how much of that wage to invest in the risky asset, in storage, and which money manager  $j \in \{1, \dots, m\}$  to hire, so as to solve:

$$\max_{j \in \{1, \dots, m\}, \theta_t \in (0,1)} w_{t-1} \cdot \left[ \gamma + \theta_t \cdot \mathbb{E}(R_t - \gamma - f_{jt}) - a_{ij} \cdot \theta_t^2 \cdot \frac{\sigma_t}{2} \right]. \quad (1)$$

The preferences of workers are mean-variance with respect to the *return* of their portfolio.<sup>2</sup> Critically, the utility of the investor  $i$  depends on the identity of manager  $j$  through the fee

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<sup>2</sup>This objective function arises under quadratic utility when the agent's risk aversion is decreasing in his initial (pre-investment) wealth endowment, namely when:

$$u(c(W), W) = c(W) - \frac{b}{W} c(W)^2,$$

$f_{jt}$  charged by  $j$  and through the manager-investor specific risk aversion parameter  $a_{ij} > 1$ , which we think of as the anxiety  $i$  experiences investing with  $j$ . As in GSV (2014), saver  $i$  sees risk as being more costly with manager  $j$ , anxiety  $a_{ij}$  as higher, the lower is the trust of  $i$  for  $j$ . Investors are less anxious when taking risk with more trusted managers, perhaps because they know them or their representatives personally, or perhaps because they are persuaded by advertisement. We thus capture lower trust of  $i$  in  $j$  by a higher value of the anxiety parameter  $a_{ij}$ .<sup>3</sup>

## 2.2 Financial Intermediation

A worker's demand for the risky asset depends on his trust for different money managers and on the fees these managers charge. At each time  $t$  savers are uniformly distributed around the unit circle. Each manager  $j$  is also located along the circle at a constant distance  $\Delta \equiv 1/m$  from the adjacent managers. The number of managers is exogenously fixed at  $m$  (we endogenize  $m$  in Section 5), and the trust of worker  $i$  in manager  $j$  is given by:

$$\frac{1}{a_{ij}} = \max(\Gamma - d_{ij}, 0), \quad (2)$$

where  $d_{ij}$  is the distance along the circle between the worker and the manager. The greater is the distance between worker  $i$  and manager  $j$ , the lower is trust and the higher is the worker's risk aversion.<sup>4</sup> Parameter  $\Gamma \leq 1$  is a measure of generalized trust in the model and captures the maximal

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where consumption is the realized investment return, i.e.  $c(W) = \check{R} \cdot W$ . This utility function avoids the unappealing feature of standard quadratic utility that the share of wealth invested in the risky asset decreases with wealth  $W$ . It is also more tractable than constant relative risk aversion, which requires lognormal returns and analytical approximations that complicate optimal fee setting by money managers.

<sup>3</sup> This formulation leads to the normative conclusion that growth of finance is socially desirable even though trust creates market power distortions. An alternative, and in our view less plausible, interpretation of the model holds that delegation solely reflects investor overconfidence in the ability of managers.

<sup>4</sup> Equation (2) describes investor trust in money managers. As a consequence,  $d_{ij}$  is zero when the money manager invests his own money, but not when a saver takes risk on his own. In fact, savers neither trust themselves nor other savers for risky investment. For simplicity, we assume that investors have zero trust (their risk aversion is infinite) with respect to homemade or non-professionally managed risk taking.



distance at which investor  $i$  is willing to delegate. If  $d_{ij} \geq \Gamma$ , the investor suffers infinite anxiety, namely  $a_{ij} = \infty$ , and so he only uses the storage technology. Two managers located at distance  $\Delta$  compete for some investors as long as  $\Gamma > \Delta/2$ . An investor located halfway between these two managers is willing to take some risk with either of them. When  $\Gamma < \Delta/2$ , investors located in the middle suffer infinite anxiety from hiring either manager. These investors do not take any risk and each manager has a small, captive, clientele. As we show below, whether generalized trust  $\Gamma$  is above or below  $\Delta/2$  has interesting implications for the effect of competition on equilibrium fees.

At time  $t$  each manager sets his fee for the generation of savers born at  $t - 1$ . This results in a profile  $\mathbf{f}_t \equiv (f_{1,t}, \dots, f_{m,t})$  of managers' fees<sup>5</sup>. Given this profile, each worker  $i$  chooses, based on his trust as described by (2), which manager to invest with and how much risky investment to undertake. The optimal policy of a worker  $i \in I_{t-1}$  is summarized by a vector  $[\theta_{ij}^*(\mathbf{f}_t)]_{j=1, \dots, m}$  that takes nonzero value only for the manager to whom the worker delegates his risky investment.<sup>6</sup> This vector is the solution of the investor's problem described in Equation (1). The optimal investment policy depends on time only through the fees  $\mathbf{f}_t$  set by managers at time  $t$ . This implies that at a fee profile  $\mathbf{f}_t$ , the profit earned by a generic money manager  $j$  from time  $t$  investment is given by:

$$\pi_{jt}(\mathbf{f}_t) = f_{jt} \cdot \left[ \int_i \theta_{i,j}^*(\mathbf{f}_t) di \right] \cdot w_{t-1}. \quad (3)$$

We consider symmetric Nash equilibria in which each manager  $j$  sets the same optimal fee  $f_t^*$  identified by the condition:

$$f_t^* = \operatorname{argmax}_{f_{jt}} \pi_{jt}(f_{jt}, f_{-jt} | f_{-jt} = f_t^*).$$

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<sup>5</sup> Hsieh and Moretti (2003) show that the income of real estate agents rises when house prices rise because commissions are fixed by a trade group. In our model, fees are endogenously set by competing intermediaries.

<sup>6</sup> In our model investors optimally invest only with their most trusted manager. Since all intermediaries manage the same asset, there is no diversification motive for hiring multiple managers. Formally, when investing with multiple managers, the anxiety of investor  $i$  is equal to  $\sum_j a_{ij} z_j$ , where  $z_j$  is the share of the overall risky portfolio  $\theta_i$  invested with manager  $j$ .

### 2.3 The Productive Sector

There are two inputs: labor and capital, available in aggregate supply  $L_t = 1$  and  $K_t$ , respectively. We assume that capital can be converted back into consumption at no cost, but Online Appendix B.2 shows that our main results hold when we relax this assumption. Inputs at time  $t$  are owned by workers (labor is owned by the young born at time  $t$ , capital is owned by the old who are born at time  $t - 1$ ) and hired by firms in competitive markets. The production technology is risky. If an individual firm hires  $k_t$  units of capital and  $l_t$  units of labor it produces:

$$F(k_t, l_t) = \varepsilon_t [k_t + A \cdot k_t^\alpha l_t^{1-\alpha}]. \quad (4)$$

In (4),  $\varepsilon_t$  is an i.i.d shock with mean  $\mathbb{E}\{\varepsilon_t\} = 1$  and variance  $\sigma$ . Uncertainty is realized at the end of period  $t$  when output is produced. The value of a firm consists of two components. The first is its value added  $\varepsilon_t \cdot A \cdot k_t^\alpha l_t^{1-\alpha}$ , where  $A$  captures the firm's total factor productivity. The second component is the capital stock  $k_t$  used in production, which the firm returns to investors undepreciated (up to the stochastic shock  $\varepsilon_t$ ).<sup>7</sup>

At time  $t$ , before the shock  $\varepsilon_t$  is realized, firms hire capital and labor. Workers are hired on the spot market and are remunerated with a deterministic equilibrium wage  $w_t$ . The remuneration of capital is risky since it fully adjusts to the realization of the shock  $\varepsilon_t$ , and is paid to the holders of the firm's financial claims. These claims are bought by savers via money managers and pay an equilibrium return  $R_t$  with expected value  $\mathbb{E}\{R_t\}$  and risk  $\sigma_t$ . The return  $R_t$  is competitively determined as a function of investment and the shock  $\varepsilon_t$ .

### 3. Equilibrium in the Money Management Sector

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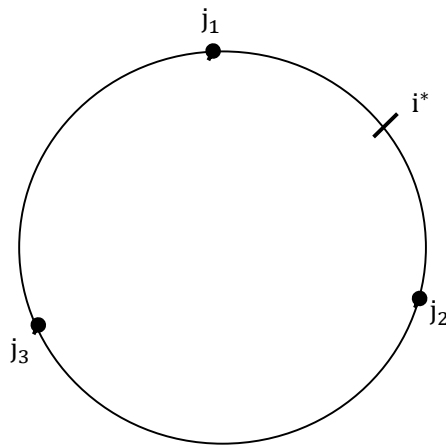
<sup>7</sup> Our results would change very little if the capital depreciation/appreciation shock was either different from the shock affecting value added or absent altogether. We can also allow for depreciation  $\delta$  of physical capital, as long as such depreciation is smaller than the waste from inefficient storage,  $\delta < 1 - \gamma$ .

To solve a worker's portfolio problem and a manager's profit maximization problem, we take wages and expected asset returns as given. These variables are computed in the next section. At time  $t$ , each saver – after collecting his period  $t - 1$  wages – optimally chooses a money manager and an amount of risky investment to solve Equation (1). If worker  $i$  selects money manager  $j$ , he invests in the risky asset a share  $\theta_{ij}(f_{jt})$  of his wealth  $w_{t-1}$ . This share is given by:

$$\theta_{ij}(f_{jt}) = \frac{\mathbb{E}(R_t - \gamma - f_{jt})}{a_{ij}\sigma_t}, \quad (5)$$

where  $\theta_{ij}(f_{jt})$  is assumed to be in  $(0,1)$  (Proposition 1 verifies that this is the case). The saver invests  $\theta_{ij}(f_{jt}) \cdot w_{t-1}$  in the risky asset and  $[1 - \theta_{ij}(f_{jt})] \cdot w_{t-1}$  in storage. Risk taking increases in the excess return paid by the risky asset and in investor trust, but decreases in the risk  $\sigma_t$  of the financial asset. Consider now a worker's decision of which money manager to hire.

Figure 2 depicts the case with three managers, in which an investor  $i^*$  is located between managers  $j_1$  and  $j_2$ . Consider the case when investors do not suffer infinite anxiety with either of the two closest managers, i.e.,  $\Gamma > \Delta/2$ .



**Figure 2**

In this situation (and focusing on small deviations from a symmetric equilibrium), the investor chooses between the two closest managers  $j_1$  and  $j_2$ . This implies that in setting his fee a generic manager, say  $j_2$ , competes for investors on his right against  $j_1$  and for investors on his left against  $j_3$ . To see the implications of this logic for fee setting, consider the general case in which an investor  $i$  chooses between his two closest managers  $j$  and  $j'$ . Denote the distance between investor  $i$  and his left-adjacent manager  $j$  by  $\delta$ . Since the total distance between the two managers is  $\Delta$ , the investor is located at distance  $\Delta - \delta$  from his right-adjacent manager  $j'$ . In light of Equation (2), these distances pin down in Equation (5) the investor's risky investment with either manager. By plugging these optimal risky investments into the investor's objective function of Equation (1), we can show that investor  $i$  obtains a higher utility by delegating his investment to manager  $j$  rather than to manager  $j'$  if and only if:

$$\delta \leq \delta(f_{jt}, f_{j't}) \equiv \Gamma - (2\Gamma - \Delta) \cdot \frac{1}{\left[ \frac{\mathbb{E}(R_t - \gamma - f_{jt})}{\mathbb{E}(R_t - \gamma - f_{j't})} \right]^2 + 1}. \quad (6)$$

Investor  $i$  thus hires manager  $j$  when the above condition holds and manager  $j'$  otherwise. Intuitively, the investor delegates his risky portfolio to manager  $j$  when his trust in  $j$  is sufficiently high, as captured by a sufficiently small distance  $\delta$  from  $j$ . Other things equal, delegation to manager  $j$  is also more likely when  $j$  charges a lower fee ( $f_{jt}$  is lower) and the competing manager  $j'$  charges a higher fee ( $f_{j't}$  is higher).

Consider now optimal fee setting by manager  $j$ . With the assumed circular structure, a generic manager  $j$  competes for investors against his neighbors on the left and the right. Manager  $j$  attracts investors who – according to (6) – are sufficiently close to him. This implies that, if two competing managers  $j'$  and  $j''$  set the equilibrium fees  $f_{j't} = f_{j''t} = f_t^*$ , then the profit of manager  $j$  from setting fee  $f_{jt}$  is given by:

$$2 \cdot w_{t-1} \cdot f_{jt} \cdot \int_0^{\delta(f_{jt}, f_t^*)} (\Gamma - \delta) \cdot \frac{\mathbb{E}(R_t - \gamma - f_{jt})}{\sigma_t} \cdot d\delta,$$

where  $\delta(f_{jt}, f_t^*)$  is the maximal distance at which an investor  $i$  prefers to hire manager  $j$  at fee  $f_{jt}$  to hiring his closest competitor at the equilibrium fee  $f_t^*$ . Maximization of the above profit function yields the (sufficient) first order condition:

$$\mathbb{E}(R_t - \gamma - 2f_{jt}) \cdot \int_0^{\delta(f_{jt}, f_t^*)} (\Gamma - \delta) \cdot d\delta + \frac{\partial \delta(f_{jt}, f_t^*)}{\partial f_{jt}} [\Gamma - \delta(f_{jt}, f_t^*)] \cdot f_{jt} \cdot \mathbb{E}(R_t - \gamma - f_{jt}) = 0.$$

At a symmetric equilibrium  $f_{jt} = f_t^*$ , we obtain the following result (all proofs are in Appendix A).

**Lemma 1** *The equilibrium fee at time  $t$  is given by:*

$$f_t^* = \left[ \frac{\Delta}{\Gamma} - \left( \frac{\Delta}{2\Gamma} \right)^2 \right] \cdot \frac{\mathbb{E}(R_t - \gamma)}{2} \equiv \varphi \cdot \mathbb{E}(R_t - \gamma). \quad (7)$$

where  $\varphi < 1$ . Management fees increase with the expected return on the risky asset. Furthermore, for  $\Gamma > \Delta/2$  – which is equivalent to  $m \geq 1/2\Gamma$ , fees decrease in the number of managers  $m$  and in the generalized trust  $\Gamma$  that investors have in the financial sector as a whole.

From the empirical standpoint, unit fees in our model correspond to the ratio between aggregate financial sector income  $f_t^* K_t$  and intermediated wealth  $K_t$ . As in GSV (2014), equilibrium fees capture a constant fraction of the excess return expected on the risky asset. This sharing rule is intuitive: managers extract part of the surplus they enable their trusting investors to access. The fraction  $\varphi$  of return extracted by managers decreases as trust in all managers  $\Gamma$  rises. When investors trust all managers, competition among them is very intense, which drives down fees. If  $m \geq 1/2\Gamma$ , fees also drop as the number of managers  $m$  rises. Intuitively, competition between highly trusted managers lowers their market power and fees. Fees fall to zero as managers

fill the entire circle, namely as  $m \rightarrow \infty$ . In the remainder, we focus on the case where  $m \geq 1/2\Gamma$ .<sup>8</sup>

We study the case  $m < 1/2\Gamma$  in our analysis of entry of Section 5.1.

By plugging Equation (7) into the optimal portfolio of Equation (5), we can show that investor  $i$  places in the risky asset a share of wealth given by:

$$\theta_{ij}(f_{jt}) = \frac{(1 - \varphi) \cdot \mathbb{E}(R_t - \gamma)}{a_{ij}\sigma_t}.$$

In equilibrium, each investor hires the closest manager and each manager attracts the same amount of wealth. As a consequence, the aggregate share of wealth invested in the risky asset at  $t$ , which we denote by  $\theta_t$ , is the product of the number of managers  $m$  and the share of wealth managed by each of them. This aggregate share is given by:

$$\begin{aligned} \theta_t &\equiv \iint_{i,j} \theta_{ij}(f_{jt}) \, didj = m \cdot 2 \cdot \left[ (1 - \varphi) \cdot \frac{\mathbb{E}(R_t - \gamma)}{\sigma_t} \cdot \int_0^{\frac{\Delta}{2}} (\Gamma - \delta) d\delta \right] = \\ &= \frac{(1 - \varphi)\mathbb{E}(R_t - \gamma)}{\sigma_t} \cdot \left( \Gamma - \frac{\Delta}{4} \right), \end{aligned} \quad (8)$$

where the expression in square brackets captures the wealth share invested by the clients to the right of a manager. With symmetry, the wealth share managed by an individual manager is twice the amount in square brackets. Equation (8) says that the share of wealth invested in the risky asset increases in the asset's excess return (net of fees) per unit of risk, in overall trust  $\Gamma$ , and in the number of managers  $m = 1/\Delta$ . As trust in money managers increases, fees drop, investors become less anxious and are willing to take more risk.

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<sup>8</sup>The case  $m < 1/2\Gamma$  has some interesting properties. When there are very few managers, a potentially large measure of investors located between any two managers does not take any risk. In this case, an entering manager could exploit monopoly (or quasi) monopoly profits by locating close to such excluded investors. In this scenario, entry of new money managers increases participation into risk taking while exerting limited (or no) downward pressure on the fees charged by existing managers.

## 4. General Equilibrium Dynamics

### 4.1 Production, Wages and Asset Returns

At time  $t$ , before observing  $\varepsilon_t$ , a firm hires labor and capital to maximize expected profits:

$$\max_{k_t, l_t} \mathbb{E}\{\varepsilon_t \cdot k_t + \varepsilon_t \cdot A \cdot k_t^\alpha l_t^{1-\alpha} - w_t l_t - R_t k_t\},$$

which are equal to total output (inclusive of both value added and the capital stock) minus factor payments. Profit maximization yields the optimality conditions:

$$(1 - \alpha)k_t^\alpha l_t^{-\alpha} = w_t,$$

$$1 + \alpha A k_t^{\alpha-1} l_t^{1-\alpha} = \mathbb{E}\{R_t\}.$$

The marginal product of labor is equated to the wage rate, and the average marginal product of capital is equated to the average (gross) return of financial assets  $\mathbb{E}\{R_t\}$ .

Because the real wage is deterministic, the firm's wage bill is also deterministic, given by  $w_t l_t = (1 - \alpha) A k_t^\alpha l_t^{1-\alpha}$ . The production function then implies that, upon the realization of a shock  $\varepsilon_t$ , the resources available to the firm's capital suppliers are  $\varepsilon_t \cdot k_t + \varepsilon_t \cdot A \cdot k_t^\alpha l_t^{1-\alpha} - (1 - \alpha) A k_t^\alpha l_t^{1-\alpha}$ . The rate of return per unit of capital in state  $\varepsilon_t$  is therefore given by:

$$R_t = \varepsilon_t + \varepsilon_t \cdot A \cdot k_t^{\alpha-1} l_t^{1-\alpha} - (1 - \alpha) A k_t^{\alpha-1} l_t^{1-\alpha}.$$

By taking the expected value of the above expression, one can immediately see that the expected return  $\mathbb{E}\{R_t\}$  is equal to the average marginal product of capital  $[1 + \alpha A k_t^{\alpha-1} l_t^{1-\alpha}]$ , as in the first order condition above. With constant returns to scale, remunerating capital with the residual of output after the wage bill is paid is consistent with optimality. Evaluated at the aggregate endowments  $K_t$  and  $L_t = 1$ , the equilibrium wage and expected return are then given by:

$$(1 - \alpha) A K_t^\alpha = w_t, \tag{9}$$

$$1 + \alpha AK_t^{\alpha-1} = \mathbb{E}\{R_t\}. \quad (10)$$

Furthermore, by using the above expression for  $R_t$  we can show that the variance of returns is equal to  $\sigma_t = \text{var}(R_t) = \sigma[1 + AK_t^{\alpha-1}]^2$ .

## 4.2 Evolution of the Financial Sector

We can now characterize the evolution of the economy. The total amount of risky investment at time  $t$ , which buys the aggregate capital stock  $K_t$ , is equal to the past aggregate wage bill  $w_{t-1}$  times the share of this wealth invested with money managers:

$$K_t = \theta_t \cdot w_{t-1}.$$

Using Equations (9), we can rewrite this equation as:

$$K_t = \theta_t \cdot (1 - \alpha)AK_{t-1}^\alpha. \quad (11)$$

By plugging equilibrium returns and variance into equation (8), we can compute the aggregate share of wealth invested in the risky asset, which is given by:

$$\theta_t = \frac{(1 - \varphi)(1 + \alpha AK_t^{\alpha-1} - \gamma)}{\sigma[1 + AK_t^{\alpha-1}]^2} \cdot \left( \Gamma - \frac{\Delta}{4} \right). \quad (12)$$

Equations (11) and (12) fully characterize the dynamics of the economy. The law of motion of the capital stock in (11) is very similar to that obtained in a standard Solow model, with the main difference that now the amount of resources invested in the economy depends, through  $\theta_t$ , on the equilibrium fees set by money managers and on the risk-return profile entailed by real investment.

In Online Appendix A we prove that, by combining (11) and (12) we obtain the following result:



**Proposition 1** *If  $2\alpha > (1 - \gamma)$ , there are two thresholds  $\bar{\sigma}$  and  $\underline{\sigma}$ , with  $\bar{\sigma} > \underline{\sigma}$ , such that, for  $\sigma \in (\underline{\sigma}, \bar{\sigma})$  the economy admits a unique nonzero steady state level of capital  $K^*$  at which individual risk taking is interior and aggregate risk taking is given by  $\theta^* < 1$ . The steady state is locally stable and displays the following properties:*

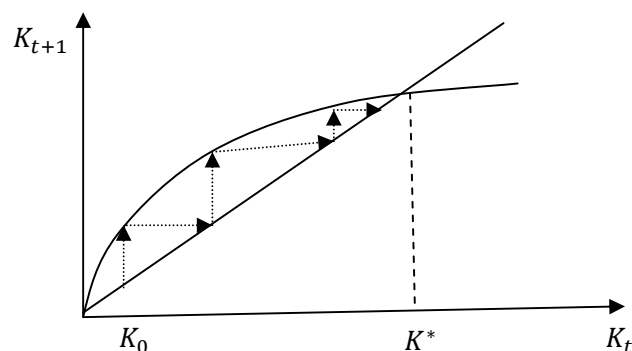
- i) The steady state capital stock weakly increases with the level of productivity and with the number of money managers, formally  $\partial K^*/\partial A > 0, \partial K^*/\partial m > 0$ ;*
- ii) Risk taking does not change with the level of productivity and increases with the number of money managers, formally  $\partial K^*/\partial A = 0, \partial \theta^*/\partial m > 0$ .*

When the volatility  $\sigma$  of the productivity shock is intermediate, the economy monotonically converges to a unique steady state level of financial intermediation and investment.<sup>9</sup> The steady state level of capital increases in productivity  $A$ . When investment becomes more productive, the wage earned by the young and the average return promised by money managers rise. Both effects increase financial intermediation, investment and output in the economy. An increase in the number  $m = 1/\Delta$  of money managers also increases financial intermediation, investment and output in the steady state. There are two reasons for this. First, when  $m$  increases, investors can find a more trusted money manager, increasing - for given fees - their propensity to invest. Second, a higher  $m$  increases competition among money managers, reducing equilibrium fees and increasing for a given level of an investor's trust the investor's risk appetite. As we show in Section 5.1, higher  $m$  also increases - on the extensive margin - the number of households taking risk.

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<sup>9</sup>The role of production risk is intuitive. If  $\sigma$  is too low, people are very eager to invest in the risky asset. Some or all of them give all of their wealth to money managers, setting  $\theta_{t-1}^{ij} = 1$ . Condition  $\sigma > \underline{\sigma}$  rules out this possibility. If  $\sigma$  is very high, the variance of the risky asset decreases very fast with the capital stock. This can be a source of multiplicity: some equilibria are characterized by low investment and high risk (which vindicates low investment), while other equilibria feature high investment and low risk (vindicating high investment). Condition  $\sigma < \bar{\sigma}$  rules out this possibility.

The steady state is locally stable: an economy starting below or above the steady state monotonically converges to it. Figure 3 graphically illustrates this convergence process.



**Figure 3**

Similarly to the standard neoclassical growth models, the main source of stability is diminishing returns to capital. As the capital stock increases, wages and national income rise. This raises the demand for financial assets by savers. The increase in financial assets further increases the capital stock and thus output next period. The growth rate of the capital stock however declines over time, because new resources are invested at progressively lower returns. Growth stops eventually and the steady state is attained.<sup>10</sup>

This convergence process has interesting implications for the financial sector. In particular, how do fees and money management profits change as the economy grows over time? We address these issues below.

**Corollary 1** *Suppose that the economy starts below the steady state, namely  $K_0 < K^*$ . During the transition to the steady state:*

- i) The unit fee charged by money managers, which is given by:*

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<sup>10</sup> Unlike in the standard Solow model, diminishing returns here are not enough to guarantee stability, because in our model risk taking by households increases as the capital stock grows. The reason is that capital deepening reduces, for any given  $\sigma$ , the variance of  $R_t$ . This phenomenon creates the possibility of explosive paths on which capital accumulation begets further risk taking and capital accumulation. The upper bound on the variance of shocks  $\sigma$  ensures stability by reducing the sensitivity of risk taking to the capital stock.

$$f_t^* = \varphi \cdot \mathbb{E}(R_t - \gamma) = \varphi \cdot [1 - \gamma + \alpha \cdot A \cdot K_t^{\alpha-1}], \quad (13)$$

decreases over time as capital accumulates.

- ii) The total income of the financial sector increases over time, at a higher speed than value added. The ratio of financial sector income over value added (GDP), is given by:

$$\frac{\varphi \cdot \mathbb{E}(R_t - \gamma) \cdot K_t}{AK_t^\alpha} = \varphi \cdot \left[ \left( \frac{1 - \gamma}{A} \right) \cdot K_t^{1-\alpha} + \alpha \right]. \quad (14)$$

As the economy accumulates capital, there are more resources for financial intermediation. At the same time, diminishing returns to physical capital ( $\alpha < 1$ ) imply that *ceteris paribus* these additional resources are employed at a lower marginal return. This explains why unit management fees fall along the transition. As capital deepening reduces the expected excess return on the risky asset, it also reduces the surplus that money managers can extract from investors.

Despite this reduction in unit fees, the aggregate income earned by money managers grows over time. This is because the growth in the size of the intermediated wealth  $K_t$  more than compensates for the reduction in unit fees, and causes financial sector income to rise over time. In our model financial sector income grows faster than value added, so the ratio of financial sector income to GDP grows over time. In Equation (14) we exclude storage from the definition of GDP because this technology simply allows a transfer of the capital stock across periods without creating new value added in any period.<sup>11</sup> To illustrate our results most starkly, we also exclude the remuneration from the definition of GDP the remuneration paid to finance for wealth preservation. This exclusion is also immaterial: after accounting for wealth preservation, the definition of GDP becomes  $\varphi(1 - \gamma) \cdot K_t + AK_t^\alpha$ , and finance still increases as a share of GDP because  $\alpha\varphi < 1$ .

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<sup>11</sup> In the text we consider the simplest case in which the elderly consume all of the current capital stock before dying. In this case, the capital stock is preserved only for one period. In Online Appendix B.2 we allow the elderly to sell their capital stock to the newborns. In this case, the capital stock is preserved for a potentially infinite period (there is no depreciation). Our predictions are not affected by the trading of the capital stock.

To understand this result, recall that in our model financial sector income can be viewed as remuneration for two services. The first is a “wealth preservation” service: money managers allow savers to access investment opportunities which on average return the initial un-depreciated capital and are thus better than self-storage. The second is a “growth” service: money managers enable savers to earn part of the capital income generated by these productive investment opportunities. In equilibrium, money managers are remunerated for both services. The remuneration for wealth preservation is equal to  $\varphi(1 - \gamma)K_t$ , which is the product of the per unit of return fee  $\varphi$  times the surplus created by managers relative to riskless storage. Intuitively, wealth preservation is more expensive the worse is the return on riskless storage (i.e., the lower is  $\gamma$ ). The remuneration for the growth service is equal to the per unit of return fee times capital income, namely  $\varphi \cdot \alpha \cdot AK_t^\alpha$ . This remuneration increases in total value added  $AK_t^\alpha$  and in the share  $\alpha$  of the value added that remunerates capital. As capital stock grows, the remuneration for both wealth preservation and growth services rises, in turn increasing the aggregate income of the financial sector.

Why does the total financial income grow faster than GDP? Consider the financial sector’s growth services and wealth preservation separately. As a product of real growth opportunities, income from growth services grows at the same rate as GDP. Indeed, as shown by the second term in Equation (14), the remuneration for growth services is a constant fraction  $\varphi \cdot \alpha$  of aggregate GDP. On the other hand, the first term in Equation (14) shows that the wealth preservation service grows with the wealth to GDP ratio, and thus with the capital to GDP ratio  $K_t/Y_t$ . Finance grows relative to GDP precisely because  $K_t/Y_t$  rises over time. In our model, this effect comes from diminishing returns: as the economy matures, the extra capital created is invested at progressively lower returns, causing the capital to GDP ratio to increase over time. The fact that a portion of the financial services is dedicated to preserving the wealth of the economy, and not to the shrinking pool of new profitable investment projects, causes the ratio of financial to total income to rise over

time. This provides a novel rationale for Philippon's (2013) and Philippon and Reshef's (2013) finding that the financial sector grows relative to GDP.

Is there empirical support for our main prediction that the finance income share should grow with the wealth to GDP ratio? Figure 4 presents the wealth to GDP ratio, computed for both total and financial wealth, for the United States, and shows that it rises over time. Piketty and Zucman (2014) show for several developed countries that the ratio of wealth to GDP indeed grows over long stretches of time, although they do not connect this finding to the growth of finance.

### 4.3 Fluctuations in the Size of the Financial Sector

We have so far focused on long term trends and have ignored fluctuations in the size of the financial sector, evident in Figure 1. Our model also allows us to analyze the short and long run responses of the financial sector to shocks. We compare the effects of two permanent shocks: a permanent drop in productivity  $A$  and a drop in the overall level of trust in the financial sector  $\Gamma$ , owing for instance to the erosion of investor confidence during a large scale financial crisis. Our model describes how the financial sector adjusts to these shocks.

**Corollary 2** *Suppose that an economy is originally in a steady state  $K^*(\Gamma, A)$ .*

- i) *Productivity  $A$  permanently drops to  $A' < A$ . On impact, at a given initial capital stock  $K^*(\Gamma, A)$  investment drops, financial intermediation drops, but financial sector income increases relative to GDP. Over time, the capital stock and intermediation decrease to the new steady state  $K^*(\Gamma, A') < K^*(\Gamma, A)$ , and financial sector income relative to GDP returns to the initial level.*
- ii) *Trust  $\Gamma$  permanently drops to  $\Gamma' < \Gamma$ . On impact, at a given initial capital stock  $K^*(\Gamma, A)$  investment and financial intermediation drop, and financial sector income*

*decreases relative to GDP. Over time, the capital stock and intermediation gradually fall to the new steady state  $K^*(\Gamma', A) < K^*(\Gamma, A)$ , and financial sector income decreases relative to GDP.*

A drop in either productivity or trust causes financial intermediation to shrink, both in the short and in the long run (at least weakly). In the short run, the two types of shocks entail different responses in the relative size of the financial sector. While a drop in productivity causes the relative size of the financial sector to increase, a drop in trust causes the relative size of the financial sector to decline. This is because the drop in productivity reduces GDP and growth opportunities a lot but leaves the wealth preservation service of the financial sector relatively unaffected. As a consequence, the financial sector shrinks less than GDP, increasing the share of national income going to finance. In contrast, a drop in trust reduces the remuneration of both the wealth preservation and growth services of the financial sector. Although such a drop also reduces investment and income, on impact it exerts a much more drastic effect on the financial sector income, causing the relative size of finance to drop.

In our model, permanent shocks to productivity or generalized trust can generate long lasting boom and bust cycles to the size of the financial sector. As trust suddenly dissipates (owing, for instance, to a financial crisis), individuals take money out of the financial sector and put it into mattresses (self-storage). This reduces financial intermediation and the financing of profitable investment opportunities. Income reductions reduce the stock of wealth, further undermining the ability to finance investment in the future. This process generates a persistent contraction in financial intermediation and income until the new, lower, equilibrium is attained.

## **5. Extensions**

## 5.1. Entry into the Financial Sector

Our analysis has so far focused on the dynamics of fees and of financial intermediaries' income as shaped by the progressive exhaustion of investment opportunities (the diminishing returns assumption). In so doing, we neglected another important dimension of financial sector evolution, namely entry of new financial intermediaries, which was precluded by the assumption that the number of money managers is fixed at  $m = 1/\Delta$ .

We now allow for endogenous entry of financial intermediaries. Formally, we allow the distance  $\Delta_t$  at time  $t$  between two adjacent money managers to fall over time due to entry. Denote the number of financial intermediaries at  $t$  by  $m_t = 1/\Delta_t$ . For notational simplicity we treat this variable as continuous, even though the number of active managers is equal to the largest integer below  $m_t$ . We assume that creating a new money management firm at time  $t$  costs  $\eta \cdot AK_t^\alpha$  units of consumption, where  $\eta > 0$ . This cost should be viewed as the value of labor that the founder must expend in order to setup the new financial intermediary and to earn the trust of investors (indeed, the opportunity cost of time at  $t$  is equal to the wage rate, which scales with value added).<sup>12</sup> Money managers can enter/exit at any time, so current profits are the only determinants of entry decisions. Finally, money managers appear in discrete and thus negligible numbers, so entry of additional managers leaves the labor supply of productive firms unchanged.

To investigate the effects of entry, we allow for the case in which initially some investors are so distant from money managers that they prefer not to take any portfolio risk. Denoting by  $m_0 = 1/\Delta_0$  the initial number of managers, this boils down to assuming that  $\Gamma < \Delta_0/2$ . In the simplest case where initially there are two managers ( $m_0 = 2$ ), the distance between them is half the circle ( $\Delta_0 = 1/2$ ), and  $\Gamma < 1/4$  ensures that investors located halfway between them suffer from

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<sup>12</sup> In this formalization, entering managers locate along the unit circle halfway between existing managers. This allows them to maximize distance from existing managers, which maximizes profits and allows all managers to be located at the same distance  $\Delta_t$ .

infinite anxiety and invest everything in storage. In this case, each manager has a captive clientele, and set fees as monopolists. As managers enter, the distance between two adjacent managers shrinks. From the first time when  $\Delta_t/2 < \Gamma$  onwards, managers start competing with each other as in the case of Sections 3 and 4.

By generalizing our previous analysis, the proof of Lemma 2 shows that equilibrium fees at time  $t$  are now given by:

$$f_t^* = \varphi(\Delta_t) \cdot \mathbb{E}(R_t - \gamma), \text{ where } \varphi(\Delta_t) \equiv \begin{cases} \frac{1}{2} & \text{if } \frac{\Delta_t}{2} > \Gamma \\ \left[ \frac{\Delta_t}{\Gamma} - \left( \frac{\Delta_t}{2\Gamma} \right)^2 \right] \cdot \frac{1}{2} & \text{if } \frac{\Delta_t}{2} < \Gamma \end{cases}. \quad (15)$$

When there are few money managers, each of them acts as a monopolist and charges a constant fee per unit of excess return. As money managers become denser in the circle and start competing with each other, the fee  $\varphi(\Delta_t)$  per unit of excess return increases in  $\Delta_t$ . In this range, competition among money managers is less intense when there are fewer managers ( $\Delta_t$  is higher).

If at time  $t$  a number  $1/\Delta_t$  of money managers is active, the total profits of the financial sector are equal to  $f_t^* K_t = \varphi(\Delta_t) \cdot [(1 - \gamma)K_t + \alpha K_t^\alpha]$ . At time  $t$ , money managers enter until the profit earned by each of them is equal to the setup cost. This condition is given by:

$$\frac{f_t^* K_t}{m_t} \equiv \varphi(\Delta_t) \cdot \Delta_t \cdot [(1 - \gamma)K_t + \alpha K_t^\alpha] = \eta \cdot AK_t^\alpha. \quad (16)$$

By dividing both sides by  $AK_t^\alpha$ , we can rewrite the equilibrium entry condition as:

$$\varphi(\Delta_t) \cdot \Delta_t \cdot [(1 - \gamma)K_t^{1-\alpha} + \alpha] = \eta. \quad (17)$$

Here  $\varphi(\Delta_t)$  captures the fee charged by each money manager per unit of service provided (be it wealth preservation or growth). This component increases with  $\Delta_t$  because a drop in the number of managers raises fees and the aggregate income of each manager.



The second term  $\Delta_t \cdot [(1 - \gamma)K_t^{1-\alpha} + \alpha]$  on the left hand side captures the share of the aggregate value of money managers' services to aggregate income provided by each individual manager at time  $t$ . As shown in the previous section, this ratio increases with the capital stock  $K_t$  because financial intermediaries' wealth preservation service becomes relatively more important as the country becomes richer. This feature drives one key property of the entry model, which we summarize in the result below.

**Lemma 2** *Consider a path along which the capital stock  $K_t$  increases over time. Equation (17) implies that along this path:*

- i) *The number of active money managers increases (i.e.,  $\Delta_t$  drops) over time.*
- ii) *The management fees charged per unit of capital fall over time, owing both to the drop in  $\varphi(\Delta_t)$  as new money managers enter, and to the fall in the marginal return to capital as  $K_t$  increases.*
- iii) *Entry of new managers boosts risk taking both on the extensive margin, as the number of risk taking households increases, and on the intensive margin.*
- iv) *The aggregate income of the financial sector increases over time, both in absolute terms and relative to the country's aggregate income.*

As the capital stock expands, there are more resources available for intermediation. For given fees, money management becomes more profitable, so incurring the setup cost  $\eta \cdot AK_t^\alpha$  becomes worthwhile. This stimulates entry of new money managers, leading to a drop in  $\Delta_t$  until the profits available to an entering money manager drop back to the setup cost. In this process, managers fill the circle and increase proximity to their clients. As proximity rises, some households exclusively relying on safe storage start taking portfolio risk. In addition, competition among

managers increases, driving fees down. This pro-competitive effect of entry adds to the downward pressure on fees caused by capital deepening.

Despite the drop in unit fees, increased risk taking implies that the aggregate profits of the financial sector increase over time. As before, the expansion in the capital stock increases the demand for financial services. This force, which increases profits, is so strong that it more than offsets the drop in fees. Financial sector income increases not only in absolute terms but also relative to GDP. In equation (17), the left hand side must stay constant, which implies that the total income share absorbed by finance  $\varphi(\Delta_t) \cdot [(1 - \gamma)K_t^{1-\alpha} + \alpha]$  increases even though higher capital stock  $K_t$  causes  $\Delta_t$  to drop.

Lemma 2 considers what happens to entry and to the size of the financial sector as the capital stock  $K_t$  grows over time. We still need to verify, however, that with endogenous entry our model delivers an increasing path for the capital stock. In this case, the law of motion of the economy is still captured by Equations (11) and (12) with the only difference that now also  $\varphi(\Delta_t)$  and  $\Delta_t$  evolve according to Equation (17). In Online Appendix A we prove the following result.

**Proposition 2** *If the parametric conditions of Proposition 1 hold, and in addition productivity  $A$  is sufficiently high, the entry model admits a unique and locally stable nonzero steady state  $K^*$ .*

Starting from initial levels of capital  $K_0$  below the steady state, the transitional growth path is characterized by capital deepening, increasing financial intermediation, rising wealth, entry of money managers, greater participation in risky investments by households, decline in fees, but also increasing financial sector income both in absolute terms and relative to GDP. A high level of  $A$  ensures equilibrium uniqueness by bounding the role of the wealth preservation service provided by the financial sector. If  $A$  and thus the return from growth services is low, a high capital stock may alone create a strong demand for financial services, generating massive entry of intermediaries in

the economy, in turn sustaining massive investment. A large  $A$  creates a sizeable demand for financial intermediation regardless of the wealth preservation component, precluding the possibility of multiple equilibria.

## 5.2 Population Growth

We now relax the assumption of constant population (i.e.,  $L_t = 1$ ) to investigate the effect of population growth on the evolution of financial income. Suppose that the number of newborns grows at rate  $n > 0$  from one generation to the next. Labor supply then satisfies the law of motion:

$$L_t = (1 + n)L_{t-1}.$$

Denote by  $\widehat{K}_t \equiv K_t/L_t$  the capital stock per worker. The real wage and the expected return to capital are respectively given by:

$$(1 - \alpha)A\widehat{K}_t^\alpha = w_t,$$

$$\mathbb{E}\{R_t\} = 1 + \alpha A\widehat{K}_t^{\alpha-1},$$

with a variance of  $\sigma_t = \text{var}(R_t) = \sigma[1 + A\widehat{K}_t^{\alpha-1}]^2$ .

To characterize the effect of  $n$ , note that the previous equations imply that the share of wealth invested in the risky asset depends on the per-worker capital stock as follows:

$$\theta_t = \frac{(1 - \varphi)(1 + \alpha A\widehat{K}_t^{\alpha-1} - \gamma)}{\sigma[1 + A\widehat{K}_t^{\alpha-1}]^2} \cdot \left(\Gamma - \frac{\Delta}{4}\right).$$

The capital stock  $K_t$  employed at time  $t$  is then equal to the risky asset share  $\theta_t$  times the total wage bill paid to workers at  $t - 1$ , namely  $K_t = \theta_t \cdot w_{t-1} \cdot L_{t-1}$ . Dividing both sides by  $L_t$ , and using the expression for  $w_{t-1}$ , we find that capital per worker evolves according to:

$$\widehat{K}_t = \frac{\theta_t}{(1+n)} \cdot (1-\alpha)A\widehat{K}_{t-1}^\alpha.$$

The only difference from the law of motion described in Equation (11) is that now the fraction of wealth invested in the risky asset is scaled down by population growth  $(1+n)$ . Several immediate consequences follow. First, the capital stock per worker monotonically converges to a nonzero steady state value  $\widehat{K}^*$  that is a decreasing function of  $n$ . In this steady state, output per worker and the extent of risk taking  $\theta_t$  are also constant.

Second, the comparative static properties described by Proposition 1 continue to hold with respect to the steady state levels of capital per worker and of the extent of risk taking. The transitional growth of finance income also does not change from Corollary 1. In particular, the management fee per unit of capital declines over time as  $\widehat{K}_t$  increases toward its steady state level and financial sector income rises faster than value added. Critically, now the steady state capital to GDP ratio (and thus the steady state finance income share) increases as population growth  $n$  falls. In this sense, declining population growth also helps account for an increasing finance share<sup>13</sup>.

## 6. Empirical Predictions.

Our model yields several empirical predictions, some consistent with the available evidence, some new. The key equation of our model is:

$$\text{finance income as a share of GDP} = f_t \theta_t \left( \frac{W_t}{Y_t} \right),$$

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<sup>13</sup> Population growth also allows us to analyze the role of rational asset price bubbles, which arise naturally in OLG models (Samuelson 1958, Tirole 1985). As we show in Online Appendix C, rational bubbles can help account for the evidence on the importance of valuation effects for the growth of wealth in Piketty and Zucman (2014), and may also help explain why the finance share has grown faster than our model predicts.

where  $W_t$  is aggregate wealth at  $t$  and  $f_t\theta_t$  is the cost of intermediation. This equation breaks down the analysis of the dynamics of the financial sector into three components: the dynamics of the wealth to GDP ratio  $W_t/Y_t$ , the dynamics of fees  $f_t$ , and the dynamics of risk taking  $\theta_t$ . Here are our main predictions concerning these components.

1. The income share going to finance increases in the wealth to GDP ratio. The wealth to GDP ratio (which is monotonic in  $K_t/Y_t$  in our basic model) increases as GDP growth decelerates, and decreases when some capital is destroyed (e.g., in wars).
2. Fees  $f_t$  for a given financial product decline with the wealth to GDP ratio.
3. As the wealth to GDP ratio rises, entry of new intermediaries induces households to reallocate their portfolios toward riskier, and thus more intermediated, assets ( $\theta_t$  goes up). This effect may increase the average fee paid to money managers.
4. Fluctuations in trust influence financial income through fees, wealth allocation to risky products, and the long run level of wealth.

Prediction 1 is due to the fall in the capital income ratio during transitional growth in our neoclassical model. As economic growth slows down, the role of wealth preservation goes up, increasing the finance income share. Prediction 1 can account for the Philippon (2013) finding of the rising finance share in the U.S. Piketty and Zucman (2014) show that part of the rise of  $K/Y$  in the U.S. and other developed economies is precisely due to the slowdown in aggregate economic growth. According to Penn World Tables, annual U.S. *per capita* real GDP growth was 2.27% during 1950-1970, 2.18% during 1970-1990, and only 1.38% during 1990-2010. Over the same periods, annual population growth was 1.49%, 0.98%, and 1.07%, respectively, so *total* GDP growth has slowed down over this period from 3.86% to 3.20% to 2.51%.

For Japan and European countries, Piketty and Zucman attribute the growth slowdown primarily to declining population growth. In our model, a decline in population growth indeed

renders diminishing returns to capital more severe, as shown in section 5.2. This effect increases the steady state capital to income ratio, thereby raising the finance income share. Another prediction concerns the role of wars. Both the U.S. and Canada experienced declines in the finance income shares during World War I and II (Philippon and Reshef 2013), but neither country experienced significant war destruction. Philippon and Reshef also present some supportive data for Belgium, Spain, and the U.K., but lack of data prevents a systematic analysis across countries<sup>14</sup>.

Prediction 2 comes from the combination of two forces: diminishing returns to capital and entry of new intermediaries. As capital accumulates, the expected return on capital declines and equilibrium fees, which are a share of expected return, decline as well. This prediction raises the question of whether expected returns have in fact declined in the data. Although both stock and bond market returns are extremely volatile, our reading of the available research suggests that real interest rates (Campbell, Shiller, and Viceira 2009) and estimated equity premia (Campbell 2008, Wachter 2013) have declined steadily and substantially in recent decades.

A second and probably more important reason for falling fees is that, as the wealth to income ratio rises, the financial sector becomes more profitable, which induces entry of new money managers. As a result of such entry, the supply of trusted money managers increases. This effect intensifies competition among money managers in fee setting, leading to a decline in unit fees for a given product. In line with this prediction, French (2008) and Greenwood and Scharfstein (2013) document the decline in management fees over time.

Prediction 3 is due to entry of new money managers. By increasing the proximity of money managers to investors, entry increases risk taking and the size of the financial sector. On the extensive margin, entry increases the number of risk-taking households. On the intensive margin, entry increases the portfolio risk taken by each household. Increasing participation into risk taking

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<sup>14</sup> Piketty and Zucman (2014) suggest that valuation effects that are absent in our basic model explain a significant share of the increase in the US wealth to GDP ratio in the past 40 years. The model of bubbles in Online Appendix C may help explain this evidence.

in turn implies that despite the reduction in the equilibrium unit fee  $f_t^*$ , the unit cost of financial intermediation may actually increase as the financial sector expands. To see this, note that the total amount of financial assets in the economy at time  $t$ , which includes both storage and risky assets, is equal to  $w_{t-1}$ , the total wealth of the elderly. At the same time, the total income absorbed by the financial sector is equal to the fee times risky investment  $f_t^* K_t = f_t^* \theta_t \cdot w_{t-1}$ , where  $\theta_t$  is the wealth share that the elderly allocate to risk taking. The unit cost of financial intermediation is then given by:

$$\frac{f_t^* \theta_t \cdot w_{t-1}}{w_{t-1}} = f_t^* \theta_t.$$

As the financial sector grows, unit fees  $f_t^*$  fall but the composition of investment shifts toward riskier assets:  $\theta_t$  rises. As we show in Online Appendix B3, the latter effect may actually dominate, causing unit costs of intermediation to rise over time.

Increased risk taking can help reconcile the French (2008) finding that in the last 30 years unit fees have come down for equity mutual funds with Philippon's (2013) evidence that the unit cost of finance have stayed roughly constant, or have even increased slightly. Some new evidence we have assembled is consistent with the increased risk taking by investors over time, which would help explain non-decreasing unit costs.<sup>15</sup> Figure 5 presents the ratio of risky assets to total financial assets in the United States since the 1950s. The figure shows a sharp rise of that ratio in the 1980s and 1990s, driven primarily by the rise in stock market valuations, but interrupted in the 2000's during the period of rapid growth of (supposedly) safe assets.

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<sup>15</sup> We emphasize the role of stock market investment, but the argument also applies to financial innovations, which are more likely to be accepted by investors when they are close to and hence more trusting of their managers. To see the logic behind this intuition, note that in our model unit fees increase in the return paid by the financial asset (see Equation (7)). Intuitively, the higher the return earned by investors, the higher the rent that the trusted money manager can extract. As a consequence, the introduction of a higher return-higher risk financial asset by a trusted money manager allows the manager to charge higher fees to investors, increasing the average cost of intermediation. We leave the formal analysis of multiple risky assets to future research.

Perhaps even more dramatically, Figure 6 presents the share of the US population holding stocks. We have compiled this figure by pulling together various sources, including McCoy (1927), Bernheim and Schneider, eds. (1935), Temporary National Economic Committee (1940), Blume, Crocket and Friend (1960), as well as NYSE and Federal Reserve. Figure 6 shows a sharp rise in the share of population holding stocks in recent decades, paralleling the growth of finance income share. The shift toward higher risk taking thus emerges as a plausible explanation of the Philippon unit cost puzzle. Indeed, the similarity between Figures 1 and 6 (the correlation between the two series is .87) also points to individual investing as the ultimate source of the growing finance share.

Prediction 4 holds that fluctuations in financial income are driven, in part, by fluctuations in trust. Coming up with causal evidence on the effects of trust on financial markets is tricky, since market fluctuations are themselves likely to affect trust, so we are looking at two-way causality. Indeed, Stevenson and Wolfers (2011) present clear evidence that trust in banks in the US declined both during the savings and loans crisis in the early 1990s, and the financial crisis of 2008. But while the evidence is not definitive, trust might help shed light on some of the features of Figure 1.

Specifically, Corollary 2 may help make sense of the one dramatic fluctuation in the size of the financial sector in the United States, namely the collapse of its income from 6 to 2 percent of GDP in the Great Depression, which took 40 years to fully reverse (Figure 1). The Great Depression in all likelihood combined a decline in productivity with a sharp decline in trust in the financial system. Corollary 2 suggests that both of these factors should have led to a progressive decline in the total amount of intermediated wealth. On the other hand, the fact that the income share going to the financial sector immediately shrunk underscores the role of the decline in trust. The dramatic evidence of the decline in stock market participation in the Great Depression in Figure 6 is also broadly consistent with declining trust in the financial system<sup>16</sup>.

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<sup>16</sup> Guiso (2010) documents a decline of trust in the financial system during the S&L crisis in the US. We have found evidence of bank deposit outflows during both this period and, on a larger scale, the Great Depression.



Malmendier and Nagel (2011) present persuasive evidence that the effects of poor market performance on investor willingness to take risk are extremely long-lasting. They interpret their findings as an effect on risk aversion, which is consistent with our idea that risk aversion is in part determined by trust. The advantages of trust as the mechanism that holds the various pieces of the model together are, first, that there exist direct measures of trust, so some of our predictions can be tested using trust data, and, second, that changes in trust have predictions for the market structure of the industry that the simple risk aversion model does not have (GSV 2014).

The role of trust is also consistent with the fact that the financial sector started to grow again only after World War II, and reached its prewar size only in the 1980s, decades after the productivity and the wealth of the US economy have substantially surpassed their pre-Depression levels. The improvements in trust over long time periods might have come from restored reputations for probity, but perhaps also from government regulation, such as deposit insurance and securities laws<sup>17</sup>. The evidence of growth in stock market participation in Figure 6 is consistent with this prediction as well. The slow decades-long return of trust enabled the financial sector to reach new heights as the wealth of the US economy expanded.

## **7. Conclusion.**

We have presented a Solow-style growth model in which the financial claims on the capital stock are managed by professionals. In that model, the size of the financial sector depends both on the economy's GDP and on its stock of capital or wealth. The model accounts for some key facts about the development of the financial sector in the last century.

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<sup>17</sup> It has been suggested to us that government regulation during the Great Depression can explain the reduction in the size of the financial sector. However, the preponderance of evidence from the US and the rest of the world shows clearly that financial regulation such as securities laws and deposit insurance is associated with stronger rather than weaker financial development (e.g., La Porta et al. 1998, 2006).

To begin, the model explains why financial sector has grown relative to GDP over time (Figure 1 from Philippon 2013). The reason is that one of the functions of finance is to preserve the existing stock of wealth, and wealth has grown over time relative to income, as one would expect along the adjustment path to the steady state. The model thus also predicts the growth of the wealth to GDP ratio over time, shown in Figure 4 and more broadly by Piketty and Zucman (2014).

Our model also seeks to reconcile the somewhat conflicting evidence on the fees and unit costs of the financial sector. French (2008) presents evidence that fees on equity mutual funds have declined over time, whereas Philippon (2013) finds no evidence of declining “unit cost” of finance. According to our analysis, an important byproduct of economic growth, entry by financial intermediaries, and reduction in fees is that investors allocate increasing shares of their wealth to intermediated financial products, rather than to self-storage. This implies that the composition of investor portfolios shifts over time to riskier, and hence more expensive, financial products. This can lead to increases in unit costs, even as fees for given products decline. In line with this view, we have presented in Figures 5 and 6 some direct evidence of increased risk-taking by households as well as of growing stock market participation.

Our model’s emphasis on trust may also help explain aspects of the volatility of the financial sector. Our approach links the sharp decline of finance in the Great Depression, and its slow recovery over the following 50 years, to the rapid decline and subsequent slow recovery of trust. Part of that recovery is exogenous, as the memory of the Great Depression recedes, but part of it is also endogenous in our model, since increases in wealth encourage entry by financial intermediaries, which creates high trust relationships.

Some see the growth of finance as an indication of problems with the market economy and the financial system. Without denying the importance of rent-seeking, agency, and other problems, our paper presents a more benign view. Finance *should* grow as an economy matures, because the preservation of wealth is an increasingly important function of the financial system.

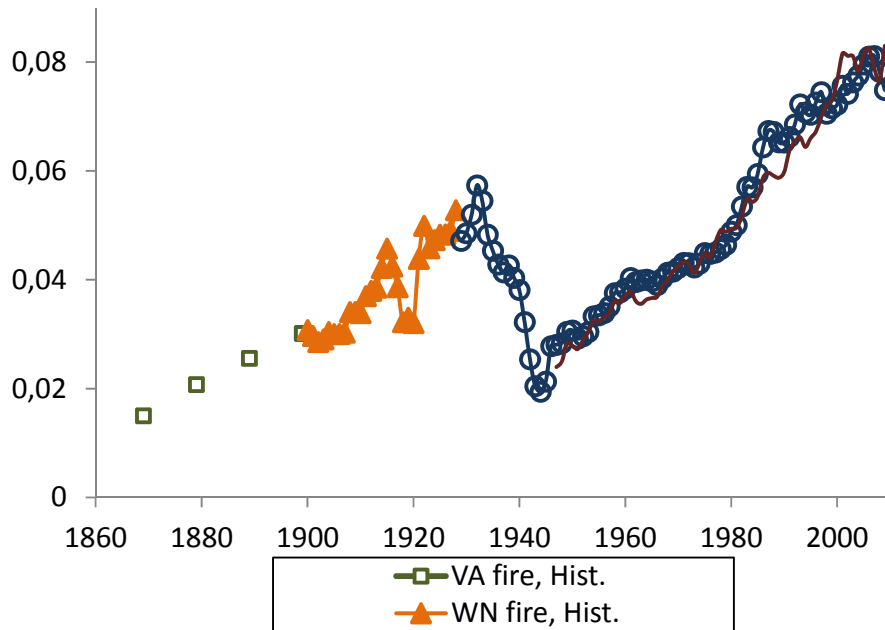
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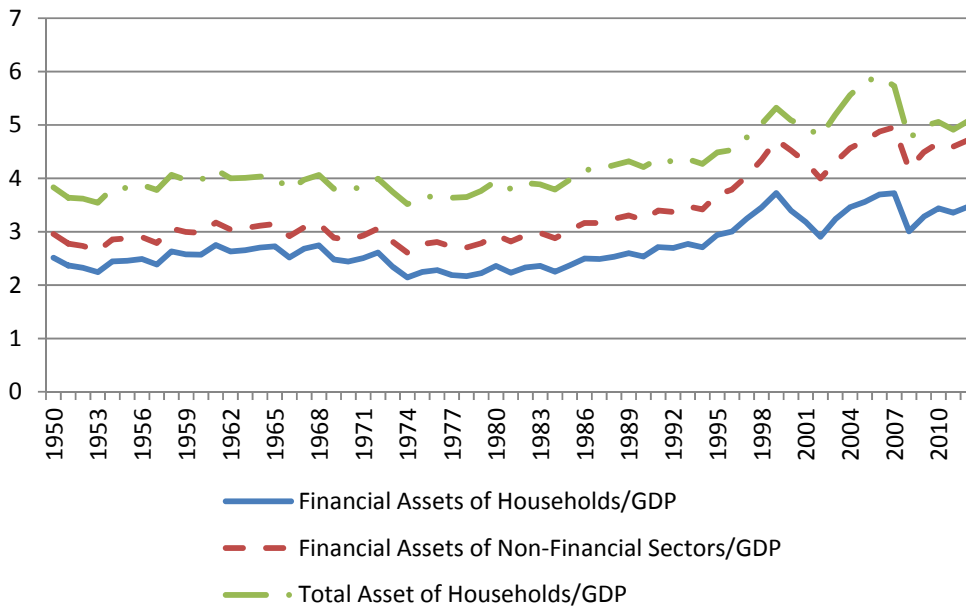
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Figure 1. Financial Sector Income/GDP



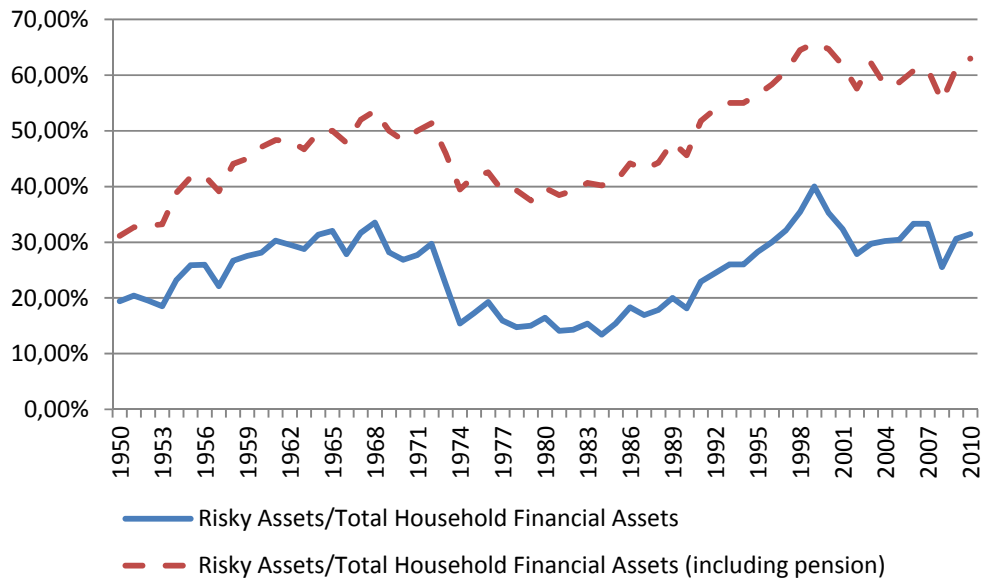
Notes: VA is value added, WN is compensation of employees, “fin” means finance and insurance, “fire” means finance, insurance, and real estate. For “NIPA”, the data source is the BEA, and for “Hist” the source is the Historical Statistics of the United States. Directly from Philippon (2013).

Figure 4. Financial Assets/GDP



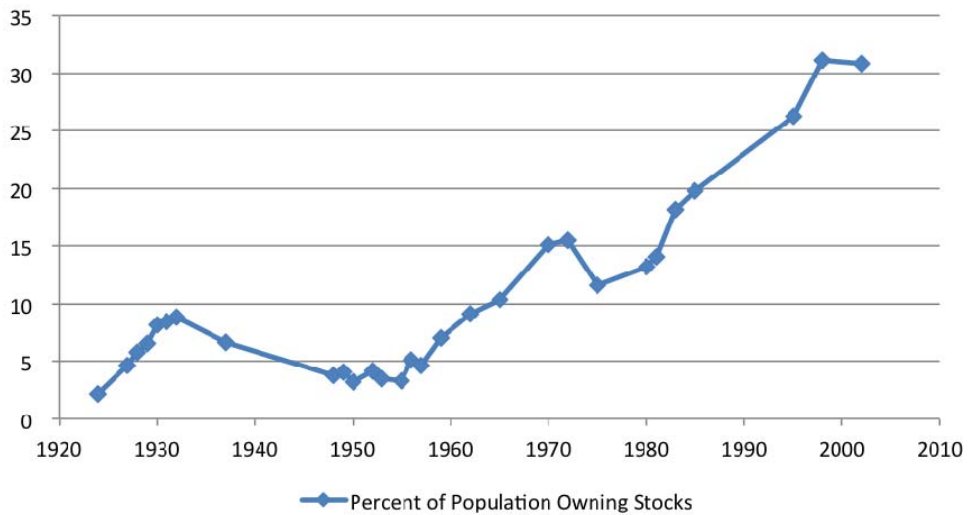
Source: Philippon (2013) and Flow of Funds. Non-financial sectors include households, nonfarm businesses.

Figure 5. Risky Asset Share in Financial Assets



Source: Flow of Funds. Risky assets include corporate equities, mutual fund shares, corporate bonds, syndicated loans, and mortgages, student loans, security credit held as assets by the household sector.

Figure 6. Share of Population Owning Stocks



Sources: McCoy (1927), Bernheim and Schneider (1935), US Congress Monograph (1940), Federal Reserve Bulletin (1949, 1950, 1951, 1954, 1958), Kimmel (1952), New York Stock Exchange Census (1956, 1959) and others.

## Online Appendix A: Proofs

**Proof of Proposition 1.** By plugging Equation (12) into (11), it is easy to see that any steady state with positive capital stock  $K^* > 0$  and such that  $\theta^* < 1$  is identified by the equation:

$$K^* = \frac{(1 - \varphi)(1 + \alpha A(K^*)^{\alpha-1} - \gamma)}{\sigma[1 + A(K^*)^{\alpha-1}]^2} \cdot \left(\Gamma - \frac{\Delta}{4}\right) \cdot (1 - \alpha)A(K^*)^\alpha,$$

which can be rewritten as:

$$c \cdot [(K^*)^{1-\alpha} + A]^2 = [(1 - \gamma)(K^*)^{(1-\alpha)} + \alpha A] \cdot A, \quad (A1)$$

where  $c \equiv \frac{\sigma}{(1-\varphi) \cdot (\Gamma - \frac{\Delta}{4}) \cdot (1-\alpha)}$ . We can verify the above equation admits a unique solution  $K^* > 0$  provided  $c < \alpha$ , which imposes an upper bound on  $\sigma$ .

Before studying the steady state, we need to verify that  $\theta^* < 1$  (for all investors). From Equation (12), the household closest to a manager invests a share of wealth:

$$\theta(K_t^{1-\alpha}) = \frac{K_t^{1-\alpha}[(1 - \gamma)K_t^{1-\alpha} + \alpha A]}{c \cdot [K_t^{1-\alpha} + A]^2} \cdot z,$$

where  $z = \frac{\Gamma}{(\Gamma - \frac{\Delta}{4}) \cdot (1-\alpha)}$ . The function  $\theta(\cdot)$  is increasing in  $K_t^{1-\alpha}$  provided  $K_t^{1-\alpha} < A$ , which as we show is strictly satisfied at the steady state capital level, and thus along transitional dynamics around the steady state. This implies that starting from a below steady state level of capital stock, risk taking increases over time until the steady state is reached. As a consequence, by exploiting Equation (A1), all investors set an interior level of risk taking at the steady state provided  $(K^*)^{1-\alpha} < A/z$ , where  $z > 1$ . By plugging this condition into (A1), we find this is equivalent to:

$$c > \frac{z \cdot [(1 - \gamma) + \alpha z]}{(1 + z)^2},$$

which imposes a lower bound on  $\sigma$ . The upper and lower bounds are mutually compatible, namely  $\frac{z \cdot [(1-\gamma) + \alpha z]}{(1+z)^2} < \alpha$ , provided  $2\alpha > (1 - \gamma)$ , which we assume to hold. This analysis thus identifies variance bounds  $\bar{\sigma}$  and  $\underline{\sigma}$ , with  $\bar{\sigma} > \underline{\sigma}$ , to which we restrict the analysis of our model.

Consider the steady state prevailing for  $\sigma \in (\underline{\sigma}, \bar{\sigma})$ . This is identified by Equation (A1). By applying the implicit function theorem, and after some algebra, one can find that:

$$\frac{d(K^*)^{1-\alpha}}{dA} \propto -\frac{-c(K^*)^{2(1-\alpha)} + cA^2 - \alpha A^2}{2c[(K^*)^{1-\alpha} + A] - (1 - \gamma)A} > 0, \quad (A2)$$

$$\frac{d(K^*)^{1-\alpha}}{dc} \propto -\frac{[(K^*)^{1-\alpha} + A]^2}{2c[(K^*)^{1-\alpha} + A] - (1 - \gamma)A} < 0, \quad (A3)$$

where both inequalities rely on the restriction  $(K^*)^{1-\alpha} < A/z$  and  $c < \alpha$ . Condition (A2) intuitively says that the steady state capital stock increases in productivity  $A$ . Condition (A3) says that the steady state capital stock increases in the number of managers (because lower  $\Delta$  reduces  $c$ ).



Consider now the dynamics of the model. By exploiting Equations (11) and (12), one can write the law of motion for our model economy as:

$$K_t^\alpha \frac{(K_t^{1-\alpha} + A)^2}{[(1-\gamma)K_t^{1-\alpha} + \alpha A]} - \frac{1}{c} AK_{t-1}^\alpha = 0. \quad (A4)$$

The above difference equation implicitly defines a function  $K_t(K_{t-1})$  whose slope is equal to:

$$\frac{dK_t}{dK_{t-1}} = \frac{\frac{1}{c} \cdot \alpha A}{K_{t-1}^{1-\alpha} \cdot \frac{(K_t^{1-\alpha} + A)^2}{[(1-\gamma)K_t^{1-\alpha} + \alpha A]} \left\{ \frac{\alpha}{K_t^{1-\alpha}} + \left( \frac{1-\alpha}{K_t^{1-\alpha} + A} \right) \frac{[(1-\gamma)K_t^{1-\alpha} + (\alpha-1+\gamma)A]}{[(1-\gamma)K_t^{1-\alpha} + \alpha A]} \right\}}$$

At the  $K_t = K_{t-1} = 0$  steady state, the above slope becomes equal to:

$$\frac{dK_t}{dK_{t-1}} = \frac{\alpha}{c} > 1,$$

Where the inequality is due to the assumption  $c < \alpha$ . Thus, the zero capital steady state is unstable, and the mapping  $K_t(K_{t-1})$  must cut the 45 degrees line at the interior steady state  $K^*$  with a slope less than one, implying that  $K^*$  is locally stable. The comparative statics of steady state capital  $K^*$  and risk taking  $\theta^*$  follow by inspection from Equation (A1) and Equation (12) in the text. In fact, higher number of money managers reduces the previously defined parameter  $c$ , increasing  $K^*$  (by A3) and  $\theta^*$  (by (A3)+(12)). On the other hand, higher productivity  $A$  increases  $K^*$  (by A2) by leaves  $\theta^*$  unaffected (because  $\theta^*$  in (12) depends on  $AK^{\alpha-1}$  which stays constant in the long run).

**Proof of Corollary 2** At the steady state capital sock  $K^*(\Gamma, A)$ , the new productivity level  $A'$  sets the wage rate, fees and intermediation at time  $t$ . In particular, investment and intermediation are pinned down by the equations:

$$K_{t+1} = \theta_{t+1} A \left( \frac{w_t}{A} \right),$$

$$\theta_{t+1}(K_{t+1}, A) = \frac{(1-\varphi)(1 + \alpha AK_{t+1}^{\alpha-1} - \gamma)}{\sigma[1 + AK_{t+1}^{\alpha-1}]^2} \cdot \left( \Gamma - \frac{\Delta}{4} \right).$$

where  $\left( \frac{w_t}{A} \right)$  is by definition invariant to changes in  $A$ , for the initial capital stock is predetermined.

Consider the effects of a change in  $A$ . The impact of such change on investment and intermediation is determined by the behavior of the ratio  $K_{t+1}/\theta_{t+1}(K_{t+1}, A)A$ . By the proof of proposition 1 we know that this ratio is an increasing function of  $K_{t+1}$  and a decreasing function of  $A$  at the steady state capital level. As a result, by the implicit function theorem, a drop in productivity reduces financial intermediation and the capital stock  $K_{t+1}$ . The relative size of the financial sector depends on the effect of the productivity change on the product  $AK_{t+1}^{\alpha-1}$ . Denote  $x \equiv K_{t+1}^{1-\alpha}/A$ . The relative size of finance increases with  $x$ . In this regard, note that the equilibrium condition  $\frac{K_{t+1}}{\theta_{t+1}(K_{t+1}, A)A} = M$ , where  $M$  is a constant, can be rewritten as:

$$A^\alpha \frac{x}{[\theta_{t+1}(1/x)]^{1-\alpha}} = M.$$

After some algebra, one can check that the left hand side of the above equation increases in  $x$ . As a result, an increase in  $A$  reduces  $x$  and thus the relative size of the financial sector, while a drop in  $A$  does the reverse. Finally, consider the long run response. One can see from the Proof of Proposition 1 and from Equation (A1), financial intermediation drops in the long run and the relative size of the financial sector remains constant.

Consider now the effect of a change in trust  $\Gamma$ . The equilibrium condition is the same as the one represented above. Because the function  $\theta_{t+1}(K_{t+1}, \Gamma)$  increases in  $\Gamma$ , higher trust increases investment and intermediation, while a drop in trust does the reverse. Accordingly, because also the function  $\theta_{t+1}(1/x, \Gamma)$  increases in  $\Gamma$ , an increase in trust on impact increases the relative size of the financial sector while a reduction in trust does the reverse. Finally, in the Proof of Proposition 1 we also establish that long run intermediation and the long run relative size of finance increase in trust.

**Proof of Lemma 2.** We studied fee setting for  $\Gamma \geq \Delta_t/2$ . Consider the case  $\Gamma < \Delta_t/2$ . Now each manager monopolizes investment by all households located at distance less than or equal to  $\Gamma$ . Under a uniform distribution, each manager attracts a measure of  $2\Gamma$  households, for a total of  $m_t 2\Gamma = \Gamma/(\Delta_t/2)$ . The remaining  $1 - \Gamma/(\Delta_t/2)$  households do not participate in risk taking.

In this setting, the optimal fee set by each monopolistic manager maximizes:

$$2 \cdot w_{t-1} \cdot f_{jt} \cdot \int_0^\Gamma (\Gamma - \delta) \cdot \frac{\mathbb{E}(R_t - \gamma - f_{jt})}{\sigma_t} \cdot d\delta,$$

which yields an optimal fee of  $f_t^* = \frac{\mathbb{E}(R_t - \gamma)}{2} \equiv \varphi \cdot \mathbb{E}(R_t - \gamma)$  where  $\varphi = 1/2$ . The wealth invested by the households participating in risk taking is equal to:

$$\begin{aligned} \iint_{i,j} w_{t-1} \theta_{ij}(f_{jt}) didj &= w_{t-1} \cdot m_t \cdot 2 \cdot \left[ (1 - \varphi) \cdot \frac{\mathbb{E}(R_t - \gamma)}{\sigma_t} \cdot \int_0^\Gamma (\Gamma - \delta) d\delta \right] = \\ &= w_{t-1} \cdot \frac{1}{\Delta_t} \cdot \frac{\mathbb{E}(R_t - \gamma)}{2\sigma_t} \cdot \frac{\Gamma^2}{2}. \end{aligned}$$

By Equation (17), as the capital stock increases (i.e.  $K_t$  goes up), there is entry of money managers. This causes  $\Delta_t$  to go down. As a result, the number of individuals participating in risk taking  $\Gamma/(\Delta_t/2)$  also increases. Individuals who were already taking risk continue to do so, and invest larger absolute amounts owing to their higher wages. If the capital stock keeps increasing, and entry of new intermediaries continues, at some point  $\Delta_t/2 < \Gamma$ . From this point onward, the equilibrium fee is the corresponding one in Equation (16). The remaining comparative statics then follow by inspection of Equations (16) and (17).

**Proof of Proposition 2** With endogenous entry, the evolution of the economy is described by the following equations:

$$K_t = \frac{(1 + \alpha AK_t^{\alpha-1} - \gamma)}{\sigma[1 + AK_t^{\alpha-1}]^2} \cdot (1 - \varphi_t) \cdot \left( \Gamma - \frac{\Delta_t}{4} \right) \cdot (1 - \alpha) AK_{t-1}^\alpha, \quad (A5)$$

$$\Delta_t \cdot \varphi(\Delta_t) \cdot \left[ \frac{(1 - \gamma)}{A} K_t^{1-\alpha} + \alpha \right] = \eta, \quad (A6)$$

for  $\Delta_t/2 > \Gamma$ , and

$$K_t = \frac{(1 + \alpha AK_t^{\alpha-1} - \gamma)}{\sigma[1 + AK_t^{\alpha-1}]^2} \cdot (1 - \varphi_t) \cdot \left(\Gamma - \frac{\Delta_t}{4}\right) \cdot (1 - \alpha)AK_{t-1}^\alpha, \quad (A5')$$

$$\Delta_t \cdot \varphi(\Delta_t) \cdot \left[\frac{(1 - \gamma)}{A} K_t^{1-\alpha} + \alpha\right] = \eta. \quad (A6')$$

for  $\Delta_t/2 < \Gamma$ . Equations (A5) and (A5') are essentially the same law of motion of the Proof of Proposition 1, with the only difference that now  $\Delta_t$  (and thus  $\varphi_t$ ) are endogenously determined in Equations (A6) and (A6'). In the spirit of the Proof of Proposition 1, we can rewrite (A5) as:

$$K_t^\alpha \frac{\sigma[K_t^{1-\alpha} + A]^2}{\left[\frac{(1 - \gamma)}{A} K_t^{1-\alpha} + \alpha\right]} \cdot \frac{1}{(1 - \varphi_t) \cdot \left(\Gamma - \frac{\Delta_t}{4}\right)} = (1 - \alpha)A^2 K_{t-1}^\alpha. \quad (A7)$$

Consider first the case where  $\Delta_t/2 < \Gamma$ . By replacing in Equation (A6) the expression for  $\varphi(\Delta_t)$  and by denoting  $s(x) \equiv \left[\frac{(1-\gamma)}{A}x + \alpha\right]$ , we can find after some algebra that

$$\left(\frac{\Delta_t}{\Gamma}\right)^2 - \frac{1}{4}\left(\frac{\Delta_t}{\Gamma}\right)^3 = \left[\frac{\eta}{\Gamma s(x)}\right],$$

where  $x \equiv K_t^{1-\alpha}$ . This equation has a unique solution for  $\Delta_t/\Gamma$  in  $(0,1)$  which we denote by  $\psi(x)$ .

By replacing the expression for  $\psi(x)$  in the expressions for  $\varphi_t$  and  $\Delta_t$  in Equation (P7), we find after some algebra that the law of motion of the economy is given by:

$$K_t^\alpha \cdot \frac{\sigma[x + A]^2}{\Gamma \cdot s(x) \left[1 - \psi(x) + \frac{\psi(x)^2}{4}\right] \cdot \left[1 - \frac{\psi(x)}{4}\right]} = (1 - \alpha)A^2 K_{t-1}^\alpha, \quad (A8)$$

Here again we have that  $x \equiv K_t^{1-\alpha}$ . The above difference equation has one trivial steady state at  $K_t = x = 0$ . A positive and unique steady state exists provided: i) the root multiplying  $K_t^\alpha$  on the left hand side above is monotonically increasing in  $x$ , ii) the value of the root at  $x = 0$  is below  $(1 - \alpha)A^2$ . The latter condition is met when the variance  $\sigma$  is sufficiently low. On the other hand, a sufficient condition for i) is that:

$$s'(x) = \frac{(1 - \gamma)}{A} \text{ is sufficiently small.}$$

Intuitively, in this case the main effect of higher  $x$  is to increase the numerator, leaving the denominator almost unaffected (also because in this case  $\psi'(x)$  stays small). When this is the case, there is a unique interior equilibrium  $K^* > 0$ . This equilibrium is locally stable (so that the capital stock monotonically converges to it) provided the slope of the implicit mapping  $K_t(K_{t-1})$  is above one at the  $K^* = 0$  steady state. One can check that this is the case provided  $A$  is sufficiently high and  $\sigma$  is above a threshold (consistent with the previous upper bound). The condition that  $\sigma$  be bounded is the same as the one required in Proposition 1, except that now the bounds are evaluated at the equilibrium number of managers prevailing when  $x = 0$  as entailed by  $\psi(0)$ . Since  $\psi(0)$  does not depend on productivity  $A$ , the assumption that  $A$  be sufficiently large can be added to ensure stability of the system. Note that when  $\psi'(0)$  is made small, the upper and lower bound will be consistent because locally entry responds slowly to changes in the capital stock, so that around

$x = 0$  the analysis does not virtually change from that with a fixed number of money managers. It is immediate to see that the same condition is sufficient for stability when  $\Delta_t/2 > \Gamma$ . The intuition is that also in this case a variant of Equation (A7) holds, except that now the fee  $\varphi_t$  is fixed. Thus, the condition that  $s'(x)$  be small is sufficient to guarantee that the  $\psi'(x)$  holding under the fixed fee assumption is small as well. Here  $\psi'(x)$  is smaller because changes in  $x$  leave the fee unchanged.

## Online Appendix B: Extensions.

### B.1 Technical Progress

We allow for productivity augmenting technological progress by assuming that the effective labor supply available at time  $t$  satisfies the law of motion:

$$L_t = (1 + n)(1 + x)L_{t-1},$$

where  $n$  is the rate of population growth and  $x$  is the rate of technical progress. Because the production function is Cobb-Douglas, this formulation of labor augmenting technical progress is equivalent to one in which productivity growth is factor-neutral and increases the value of  $A$ .

Denoting by  $\widehat{K}_t \equiv K_t/L_t$  the capital stock per unit of effective labor, the competitive remunerations of a unit of effective labor and of a unit of capital are respectively given by:

$$(1 - \alpha)A\widehat{K}_t^\alpha = w_t,$$

$$\mathbb{E}\{R_t\} = 1 + \alpha A\widehat{K}_t^{\alpha-1},$$

and where the variance of the return to capital is equal to  $\sigma_t = \text{var}(R_t) = \sigma[1 + A\widehat{K}_t^{\alpha-1}]^2$ . The share of wage income invested into risky asset also depends on  $\widehat{K}_t$ , namely:

$$\theta_t = \frac{(1 - \varphi)(1 + \alpha A\widehat{K}_t^{\alpha-1} - \gamma)}{\sigma[1 + A\widehat{K}_t^{\alpha-1}]^2} \cdot \left(\Gamma - \frac{\Delta}{4}\right).$$

The total value  $K_t$  of the capital stock created at  $t$  is equal to  $K_t = \theta_t \cdot w_{t-1} \cdot L_{t-1}$ . Thus, the law of motion of the capital stock per unit of effective labor is given by:

$$\widehat{K}_t = \frac{\theta_t}{(1 + n)(1 + x)} \cdot (1 - \alpha)A\widehat{K}_{t-1}^\alpha.$$

In light of the previous analysis, several immediate consequences follow. First, the capital stock per unit of effective labor converges to a nonzero steady state value  $\widehat{K}^*$  that is a decreasing function of  $n$  and  $x$ . In this steady state, the per-capita capital stock and per capita output grow at a constant rate  $x$ , while the extent of risk taking  $\theta_t$  converges to a constant. The comparative statics properties described by Proposition 1 continue to hold with respect to the steady state levels of the per capita capital stock and of the extent of risk taking. Second, the properties of evolution of the financial sector also do not change from Corollary 1. The management fee per unit of capital declines over time as  $\widehat{K}_t$  increases toward its steady state level. As a consequence, financial sector income rises faster than value added if we express both the numerator and the denominator in per effective units of labor. Finally, the qualitative properties of Corollary 2 also hold in this modified model. In sum, population and productivity growth introduce additional reasons for the growth of

the absolute size and profits of the financial sector, but do not affect the qualitative behavior of scaled variables such as unit fees and the income share going to finance.

## B.2 Trading and Valuation of the Capital Stock

In our baseline model consumption and capital are the same good, so that the elderly consume the capital stock they own at the end of their lives. This assumption simplifies the analysis, but it raises the issue of whether our results are robust to the more realistic setting in which capital cannot be converted back into consumption and so the elderly must sell their capital stock to the young. To shed light on this issue, suppose now that the consumption can be transformed into capital but capital cannot be converted back into consumption. This implies that at time  $t$  the elderly of the generation born at time  $t - 1$  must sell the economy's capital stock to the current young generation. The amount of capital held by the elderly at the end of time  $t$  is equal to  $\varepsilon_t \cdot K_t$ . If the price of capital in terms of consumption is  $p_t$ , the value at time  $t$  of the supply of capital in terms of consumption goods is equal to  $p_t \cdot \varepsilon_t \cdot K_t$ . On the demand side, the consumption income available to the young born at time  $t$  to buy – through money managers – the entire capital stock from the elderly is equal to  $\theta_{t+1} \cdot w_t$ . Of course, the young only demand capital from the elderly if the price of existing capital is not higher than the resource cost of creating new capital, i.e. provided  $p_t \leq 1$ , which importantly affects equilibrium prices.

To find the equilibrium price  $p_t$ , we must determine whether the capital stock  $\varepsilon_t \cdot K_t$  available at time  $t$  is below or above the desired investment  $\theta_{t+1} \cdot w_t$  by the young born at  $t$ . If the young wish to increase the stock of capital, namely  $\varepsilon_t \cdot K_t < \theta_{t+1} \cdot w_t$ , the equilibrium price of capital settles at  $p_t = 1$  so as to make savers indifferent between buying existing capital goods and creating new ones. If instead the young wish to reduce the stock of capital, namely  $\varepsilon_t \cdot K_t > \theta_{t+1} \cdot w_t$ , then the new capital goods will not be produced and the price drops to  $p_t = \frac{\theta_{t+1} \cdot w_t}{\varepsilon_t \cdot K_t} < 1$  so as to equate the values of the demand and the supply of capital goods.

Because our main results focus on transitions occurring below the steady state, let us consider the implications of this analysis for changes in the valuation of capital markets during these transitions. Recall that in these transitions, the desired capital stock increases over time, namely  $K_{t+1} = \theta_{t+1} \cdot w_t > K_t$ . As a consequence, if the potential shocks  $\varepsilon_t$  are sufficiently small that below the steady state capital the condition  $\varepsilon_t \cdot K_t < \theta_{t+1} \cdot w_t$  holds (at least when  $K_t$  is far enough from the steady state), then during the transitional growth phase the unit price of capital stays constant at  $p_t = 1$ . In each period, the elderly sell their capital  $\varepsilon_t \cdot K_t$  to the young, who add extra investment to implement their desired capital stock  $\theta_{t+1} \cdot w_t$ . The ex-post shock  $\varepsilon_t$  affects consumption by the elderly and new investment by the young, but leaves the aggregate capital stock next period unaffected. The law of motion of the economy is then identical to Equation (11): the possibility to trade capital goods does not affect how the economy converges to the steady state.

The possibility of trading in capital goods, however, affects the interpretation of our results. In particular, the capital stock  $K_t$  can now be interpreted as the market valuation of the aggregate wealth of the economy. The fact that the income share of the financial sector rises with  $K_t$  can then be viewed as the product of increasing capital market valuations. It should be noted, however, that in our model these valuations rise through the extensive margin – as new investment takes place – and not through increases in their unitary valuation  $p_t$ , which remains constant at 1.

## B.3: Competitive Entry of Intermediaries and the Growth of Financial Sector Income

We now show that it is possible that the unit cost of finance (the ratio of financial sector income over financial assets):

$$f_t^* \theta_t = \varphi_t(\Delta_t) \cdot (1 - \varphi_t(\Delta_t)) \cdot \left( \Gamma - \frac{\Delta_t}{4} \right) \cdot \frac{(1 + \alpha A K_t^{\alpha-1} - \gamma)^2}{\sigma [1 + A K_t^{\alpha-1}]^2},$$

may increase over time, as new intermediaries enter the market. To see why this may be the case, note that during transitional growth, the capital stock  $K_t$  increases while the distance between managers  $\Delta_t$  decreases. As a result, a sufficient condition for the product  $f_t^* \theta_t$  to increase over time is that the terms that are functions of  $\Delta_t$  decrease in  $\Delta_t$  while ratio which is a function of  $K_t$  increases in  $K_t$ . It is immediate to see that the ratio on the right increases in  $K_t$  provided  $\alpha < 1 - \gamma$ . On the other hand, one can find values such that the first term (which is a polynomial of degree 5) decreases in  $\Delta_t$  (e.g.  $\Delta_t$  close to  $\Gamma$ ). It is beyond the scope of this analysis to evaluate under what exact conditions unit costs may be increasing, but it seems that – given that  $\Delta_t$  is pinned down by  $\eta$  – one may be able to find economies (values of  $\eta$  and of the initial capital stock) for which the equilibrium  $\Delta_t$  is indeed close to  $\Gamma$  and unit costs increase over time until the steady state is reached.

### Online Appendix C: Bubbles

Suppose now that newborns can take financial risk not only by investing in the economy's capital stock, but also in a non-fundamental “bubbly” asset. It is easiest to think of this assets as just a risky pyramid scheme. A newborn buying one unit of this asset at  $t$  is entitled to receive a payment next period equal to his pro-rata share of the total market value of the same asset at  $t + 1$ . The future value of the bubble is uncertain at  $t$  because of volatility in agents' beliefs about the bubble's future value. Similarly to physical capital, then, the bubble is a risky investment that requires delegation to a trusted intermediary.

Suppose that the aggregate value of the bubble bought by newborns at  $t$  is equal to  $B_t$ . Then each newborn at  $t$  spends on the bubble an amount equal to  $b_t = B_t/L_t$ . If at  $t + 1$  the aggregate value of the bubble is  $b_{t+1}L_{t+1}$ , each of the now elderly receives from the  $L_{t+1}$  newborns an amount of consumption equal to  $b_{t+1}(L_{t+1}/L_t) = b_{t+1}(1 + n)$ . The return from purchasing the bubble for an agent born at time  $t$  is thus equal to  $(b_{t+1}/b_t)(1 + n)$ . As of time  $t$ , the expected gross return from investing in the bubble is then equal to:

$$\frac{\mathbb{E}(b_{t+1})}{b_t} (1 + n).$$

The investor's net return subtracts from the above expression the management fee.

The expectation  $\mathbb{E}(b_{t+1})$  depends on the process governing agents' beliefs. This process also pins down the risk entailed in the bubbly investment. For simplicity and to illustrate the basic idea, we assume that, at any  $t$ , newborns believe that the future value of the bubble is perfectly positively correlated with the future productivity of capital and that the variance of the return on the bubble equals the variance of the return to capital. This assumption captures the idea that the bubble effectively reflects an overvaluation of some firms in the economy, so that it co-moves with the fundamental value of capital. This formulation greatly simplifies the analysis because it implies imply that the bubble and the capital stock are perfect substitutes for the purpose of risk taking.

In particular, in equilibrium the expected return on the bubble is equalized to that on physical capital, managers charge the same fee on the two assets, and newborns select how much

overall risk to take. The portfolio shares on the bubbly asset and on the capital stock are then endogenously determined by the market value of these assets. In this case, the laws of motion of the capital stock per effective unit of labor and of the bubble satisfy the following equations:

$$\frac{\mathbb{E}(b_{t+1})}{b_t}(1+n) = 1 + \alpha \cdot A \cdot \widehat{K}_t^{\alpha-1}, \quad (C1)$$

$$\widehat{K}_{t+1}(1+n) = \theta_{t+1} \cdot (1-\alpha)A\widehat{K}_t^\alpha - b_t. \quad (C2)$$

Equation (C1) states that the expected return on the bubble is equal to the expected return on capital; Equation (C2) shows how the bubble crowds out some real investment.

To illustrate the impact of the bubble on finance income, we focus on the steady state  $(b^*, \widehat{K}^*)$ . The steady state is described by an expected value  $b^*$  around which the per worker bubble fluctuates, and an expected value  $\widehat{K}^*$  around which capital per worker fluctuates. These values are pinned down by the system of equations:

$$\alpha \cdot A \cdot (\widehat{K}^*)^{\alpha-1} = n,$$

$$b^* = \theta^* \cdot (1-\alpha) \cdot A \cdot (\widehat{K}^*)^\alpha - \widehat{K}^*(1+n),$$

subject to the condition  $b^* > 0$ , which is necessary for the existence of positive bubbles.

**Proposition 3** *There exist two thresholds  $\underline{n}$  and  $\bar{n}$ , where  $\underline{n} < \bar{n}$ , such that for  $n \in (\underline{n}, \bar{n})$  there exists a bubbly steady state  $(b^*, \widehat{K}^*)$  with  $b^* > 0$ , in which:*

- i) *The capital stock is smaller and the return to capital is higher than in the bubble-less equilibrium of Section 5.2.*
- ii) *The finance income share  $\varphi \cdot (1+n-\gamma) \cdot \frac{(\widehat{K}^*+b^*)}{A \cdot (\widehat{K}^*)^\alpha}$  is larger than in the bubble-less equilibrium of Section 5.2.*

**Proof:** A bubble-less equilibrium is identified by a per capita capital stock level  $\widehat{K}_{nb}$  satisfying:

$$\widehat{K}_{nb}(1+n) = \theta_{nb} \cdot (1-\alpha) \cdot A\widehat{K}_{nb}^\alpha.$$

A bubbly equilibrium is identified by a vector  $(b^*, \widehat{K}^*)$  satisfying the system of equations:

$$\alpha \cdot A \cdot (\widehat{K}^*)^{\alpha-1} = n,$$

$$b^* = \theta^* \cdot (1-\alpha) \cdot A \cdot (\widehat{K}^*)^\alpha - \widehat{K}^*(1+n),$$

subject to the condition  $b^* > 0$ . By plugging the equilibrium condition  $\widehat{K}^* = (\alpha \cdot A/n)^{1/(1-\alpha)}$  in the equation for  $b^*$  we find that the equilibrium admits a positive bubble if and only if:

$$\left(\frac{1-\varphi}{\sigma}\right) \left(\Gamma - \frac{\Delta}{4}\right) \alpha(1-\alpha) > \frac{1+n}{n} \frac{(\alpha+n)^2}{(1+n-\gamma)}.$$

After some algebra, one can check that under the condition  $2\alpha > (1-\gamma)$ , the left hand side of the above expression is U-shaped in  $n$ . But then, since the left hand side diverges both for  $n \rightarrow 0$  and

for  $n \rightarrow \infty$ , there are two thresholds  $n_*$  and  $n^*$ , where  $n_* < n^*$ , such that a bubbly equilibrium exists if and only if  $n \in (n_*, n^*)$ . Note that when  $n > n_*$  the economy is dynamically inefficient, in the sense that  $\alpha A \widehat{K}_{nb}^{\alpha-1} < n$ .

Finance income is higher in the bubbly than in the bubble-less equilibrium if and only if:

$$\varphi \cdot (1 + n - \gamma) \cdot \frac{\widehat{K}^* + b^*}{A \cdot (\widehat{K}^*)^\alpha} > \varphi \cdot (1 + \alpha A \widehat{K}_{nb}^{\alpha-1} - \gamma) \cdot \frac{\widehat{K}_{nb}}{A \widehat{K}_{nb}^\alpha}.$$

Given that when the bubble exists we have that  $\alpha A \widehat{K}_{nb}^{\alpha-1} < n$ , a sufficient condition for the bubble to expand financial income is that:

$$\frac{\widehat{K}^* + b^*}{A \cdot (\widehat{K}^*)^\alpha} > \frac{\widehat{K}_{nb}}{A \widehat{K}_{nb}^\alpha} \Leftrightarrow \theta^* \cdot (1 - \alpha) - \frac{n}{A} (\widehat{K}^*)^{1-\alpha} > \theta_{nb} \cdot (1 - \alpha) - \frac{n}{A} \widehat{K}_{nb}^{1-\alpha}.$$

Given that  $\widehat{K}^* < \widehat{K}_{nb}$ , a sufficient condition for the above inequality is that the bubble encourages risk taking, namely that  $\theta^* > \theta_{nb}$ . It is easy to see that this condition holds provided the increase in expected returns caused by the bubble more than offsets the increases risk  $\sigma_t$  (where the latter effect occurs because the marginal product of capital, and thus its fluctuations, increase with the bubble). A sufficient condition for  $\theta^* > \theta_{nb}$  to hold is that risk taking:

$$\theta = (1 - \varphi) \cdot \left( \Gamma - \frac{\Delta}{4} \right) \alpha^2 \frac{(1 + y - \gamma)}{\sigma[\alpha + y]^2}$$

Increases with the marginal product of capital in value added  $y = \alpha \cdot A \cdot K^{\alpha-1}$ . This is indeed the case provided  $\alpha \cdot A \cdot K^{\alpha-1} < \alpha - 2(1 - \gamma)$ . But then, given that the highest marginal return of capital is attained at the bubbly steady state, a sufficient condition for  $\theta^* > \theta_{nb}$  to hold is that  $n < n^{**} \equiv \alpha - 2(1 - \gamma)$ . It is easy to see that  $n^{**} > n_*$ . By defining  $\underline{n} \equiv n_*$  and  $\bar{n} \equiv \min(n^{**}, n^*)$ , we can see that for  $n \in (\underline{n}, \bar{n})$  the properties of Proposition 3 are verified.

As in the Samuelson and Tirole models, the bubble crowds out productive capital and raises the rate of return delivered by all financial assets. The bubble exists only if the economy is dynamically inefficient, which is guaranteed by the condition  $n > \underline{n}$ .<sup>18</sup> Population growth cannot however be too large (i.e.  $n < \bar{n}$ ), for otherwise the returns of the capital stock and of the bubble would be too volatile, and individuals would be unwilling to hold the bubble.

The bubble expands the finance income share relative to the equilibrium without bubbles of Section 5.2, for two reasons. First, the bubble raises rates of return paid by all risky financial assets. This effect increases the unit fee that money managers can charge to their clients, and thus the total income earned by financial intermediaries. Second, the risky bubble constitutes an intermediated investment that crowds out productive capital. This effect reduces per capita income below the no-bubble equilibrium level, increasing the wealth income ratio and the finance share in income.

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<sup>18</sup> Formally, this occurs when in the bubble-less equilibrium of Section 5.2 the steady state return to capital is below the population growth rate, namely  $A \cdot \widehat{K}^{\alpha-1} < n$ .